UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2011/2012

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS III
COURSE CODE	:	BWM 22003
PROGRAMME	:	2BDC
EXAMINATION DATE	:	JANUARY 2012
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER ALL QUESTIONS. B) ALL CALCULATIONS AND ANSWERS MUST BE IN THREE (3) DECIMAL PLACES.

THIS EXAMINATION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) Given the function $f(x,y)=\cos(xy)+e^{x^2y}$. Find f_x, f_y, f_{xx}, f_{yy}, and f_{xy}.

(10 marks)

(b) Use the double integral to find the surface area of the portion of the paraboloid $z=4-x^2-y^2$ that lies between $z=0$ and $z=3$.

(10 marks)

Q2 (a) Use the spherical coordinate to evaluate $\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{2-z^2}}^{\sqrt{2-z^2}}\int_{0}^{\sqrt{2-x^2-z^2}} y\,dy\,dx\,dz$

(10 marks)

(b) Use the cylindrical coordinate to find the volume of a solid G, where G is a solid bounded by the sphere $x^2+y^2+z^2=4$ and inside the cylinder $x^2+y^2=1$.

(10 marks)

Q3 (a) A particle starts at rest on a smooth inclined plane. At the end of t seconds, the position of the particle is given by

$$x(t)=\frac{-g}{4w^2}\left(e^{wt}+\sin wt\right)$$

where $g=9.81$ m/s^2 is the constant of gravity.

Suppose that the particle has moved $x=0.5$m in $t=1$s.

(i) Find the value of w in the equation above by taking the interval [-0.7, -0.6] using Secant Method.

(ii) Find the absolute error for answer that you obtained in (i) if the exact answer is w = -0.653.

(10 marks)

(b) Solve the following system of equations by using Thomas Algorithm.

$$\begin{aligned} x_1 + 2x_2 &= 9 \\ 6x_1 + 6x_2 - 8x_3 &= 1 \\ -3x_2 + x_3 &= 0 \end{aligned}$$

(10 marks)

Q4 (a) Find the smallest eigenvalue for the matrix A below by using Inverse Power Method with $v^{(0)} = (0 \quad 1 \quad 0)^T$.

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 6 & -2 \\ 0 & -1 & 3 \end{pmatrix}$$

(10 marks)

(b) Given integral $\int_1^b \frac{e^{2x}}{x^5+1}\,dx$ with step size $h = 0.25$ and $n = 8$ subintervals.

(i) Find the value of b.

(ii) Calculate the approximate value for the integral by using a suitable Simpson's rule.

(10 marks)

Q5 (a) The initial value problem

$$y' - 1 = \frac{y}{t}, \qquad \text{for } 1 \le t \le 2, \quad \text{with } y(1) = 2$$

has an exact solution $y(t) = t\ln t + 2t$.

With $h = 0.25$, approximate the solution of $y(1.25), y(1.5)$ and $y(1.75)$ by using classical Fourth-order Runge-Kutta method. Compare the result to the actual values by finding the absolute error.

(10 marks)

(b) Given the boundary value problem $x'' + 4x = \sin t$, $0 \le t \le 1$, with condition $x(0) = 0$ and $x(1) = 0$. Find the solution by solving the system of linear equations (in matrix-vector form) you derived using finite-difference method. Use $\Delta t = h = 0.25$.

(10 marks)

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$\cos 2\alpha = 1 - 2\sin^2 \alpha$ $\qquad$ $\cos 2\alpha = 2\cos^2 \alpha - 1$

Surface Area $\iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1} \; dA$

Cylindrical coordinates $\iiint_G f(x,y,z)dV = \int_{\theta=\theta_0}^{\theta=\theta_1} \int_{r=r_0}^{r=r_1} \int_{z=z_0}^{z=z_1} f(r,\theta,z)\, dz\, r\, dr\, d\theta$

Spherical coordinates $\iiint_G f(x,y,z)\, dV = \int_{\theta=\theta_0}^{\theta=\theta_1} \int_{\phi=\phi_0}^{\phi=\phi_1} \int_{\rho=\rho_0}^{\rho=\rho_1} f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

Secant method for nonlinear equations $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$

Thomas algorithm for system of linear equations:

i	**1**	**2**	**...**	***n***
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

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Numerical Integration:

Simpson's $\frac{1}{3}$ rule: $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i \right]$

Simpson's $\frac{3}{8}$ rule:

$$\int_a^b f(x)dx \approx \frac{3}{8}h\left[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + \ldots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \ldots + f_{n-6} + f_{n-3})\right]$$

Power Method for eigenvalue: $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,\ldots..$

Initial value problems for ordinary differential equation

Fourth-order Runge-Kutta method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems for ordinary differential equation:

Finite difference method:

$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}$ $\quad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$