UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2011/2012

COURSE NAME	:	ENGINEERING STATISTICS
COURSE CODE	:	BWM 20502 / BSM 2922
PROGRAMME	:	1 BEE 2 BDD/ BEE/ BFF 3 BDD/ BEE/ BFF 4 BDD/ BEE
EXAMINATION DATE	:	JANUARY 2012
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **SEVEN** (7) PAGES

Q1 (a) The discrete random variable X has probability distribution function as given in **Table Q1**.

Table Q1 : Probability Distribution Function of X

X	-1	0	1	2	3
P(X=x)	1/5	d	1/10	d	1/5

where d is a constant.

(i) Find the value of d.

(ii) Calculate E(X) and V(X).

(iii) Given the random variable $Y = 6 - 2X$, Find E(Y), V(Y) and P($X > Y$).

(10 marks)

(b) A continuous random variable X has probability density function as defined below :

$$f(x) = \begin{cases} kx^2 & , \quad 1 \le x \le 4, \\ 0 & , \quad \text{others}. \end{cases}$$

(i) Find the value of k.

(ii) Calculate P($2 < X < 3$).

(iii) Obtain mean and standard deviation for X.

(10 marks)

Q2 (a) R. H. Bruskin Associates Market Research found that 40% of Malaysians do not think that having a college education is important to succeed in the business world. If a random sample of 10 Malaysians are selected, find the probability that ;

(i) exactly 2 people will agree with this statement.

(ii) at most 3 people will agree with this statement.

(iii) at least 4 people will agree with this statement.

(iv) fewer than 5 people will agree with this statement.

(10 marks)

(b) An electronic office contains 3000 electronic components. Assume that the probability that each component was operated without failure during the useful life of the product is 0.7, and assume that the components fail independently. Approximate the probability that 2150 or more of the original 3000 components fail during the useful life of product.

(10 marks)

Q3 A tire manufacturer is interested in testing the fuel economy for two different tread patterns. Tires of each tread type are driven for 1000 miles on each of 10 different cars. The mileages, in mi/gal, are presented in **Table Q3**.

Table Q3 : Mileages for Tires for Tread 1 and 2

Car	**Tread 1**	**Tread 2**
1	24.1	20.3
2	22.3	19.7
3	24.5	22.5
4	26.1	23.2
5	22.6	20.4
6	23.3	23.5
7	22.4	21.9
8	19.9	18.6
9	27.1	25.8
10	23.5	21.4

(a) Find a 95% confidence interval for the mean of Tread 1.

(6 marks)

(b) Find a 95% confidence interval for the variance of Tread 2.

(6 marks)

(c) Find a 95% confidence interval for the ratio of variance Tread 1 and variance Tread 2.

(8 marks)

Q4 (a) As a part of the quality-control program for a catalyst manufacturing line, the raw materials (alumina and a binder) are tested for purity. The process requires that the purity of the alumina to be greater than 85%. A random sample from a recent shipment of alumina yielded the following results (in %).

93.2 87.0 92.1 90.1 87.3 93.6

A hypothesis test will be done to determine the acceptance of the shipment.

(i) State the appropriate null and alternate hypothesis.

(ii) Perform the appropriate hypothesis at 5% level of significance.

(iii) Should the shipment be accepted? Explain.

(10 marks)

(b) Two types of mango juice are surveyed in a market research. The consumption of the type 1 mango juice is normally distributed with $N(\mu_1, \sigma^2)$, while the type 2 mango juice is normally distributed with $N(\mu_2, \sigma^2)$. A sample of type 1 mango juice is selected with a sample size of 30 has a mean of 55.3 and a standard deviation of 3.7. A sample of type 2 mango juice is chosen with sample size of 35 has a mean of 67.2 and a standard deviation of 3.2.

(i) What is the probability that the mean consumption of type 1 mango juice is more than 50?

(ii) What is the probability that the difference means consumption of type 2 and type 1 mango juices is more than 10?

(10 marks)

Q5 The carbonation level of a soft drink beverage is affected by the temperature of the product. Twelve observations were obtained and the resulting data are shown in **Table Q5**.

Table Q5 : Data for Carbonation and Temperature

Carbonation Level, y	**Temperature, x (°C)**
2.60	31.0
2.40	31.0
17.32	31.5
15.60	31.5
16.12	31.5
5.36	30.5
6.19	31.5
10.17	30.5
2.62	31.0
2.98	30.5
6.92	31.0
7.06	30.5

(a) Using least square method, find the least squares line.

(7 marks)

(b) Estimate the carbonation level when the temperature is 32°C.

(2 marks)

(c) Test the null hypothesis that the slope is zero at 0.01 level of significance and describe your conclusion.

(6 marks)

(d) Find the coefficient of determination and interpret the result.

(3 marks)

(e) Find the Pearson correlation and describe your answer.

(2 marks)

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Formula

Random variables:

$$\sum_{i=-\infty}^{\infty} P(x_i) = 1, \quad E(X) = \sum_{\forall x} x \cdot P(x), \quad E(X^2) = \sum_{\forall x} x^2 \cdot P(x), \quad \int_{-\infty}^{\infty} f(x)\,dx = 1,$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot P(x)\,dx, \quad E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot P(x)\,dx, \quad Var(X) = E(X^2) - [E(X)]^2.$$

Special Probability Distributions :

$$P(x=r) = {}^nC_r \cdot p^r \cdot q^{n-r}, r = 0, 1, \ldots, n, \; X \sim B(n,p), \; P(X=r) = \frac{e^{-\mu} \cdot \mu^r}{r!}, \; r = 0, 1, \ldots, \infty,$$

$$X \sim P_0(\mu), \; Z = \frac{X-\mu}{\sigma}, \; Z \sim N(0,1), \; X \sim N(\mu, \sigma^2).$$

Sampling Distributions :

$$\bar{X} \sim N(\mu, \sigma^2/n), \; Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1), \; T = \frac{\bar{x}-\mu}{s/\sqrt{n}}, \; \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right).$$

Estimations :

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2, \quad \bar{x} - Z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}} < \mu < \bar{x} + Z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}}, \quad \bar{x} - t_{\alpha/2,v}\sqrt{\frac{s^2}{n}} < \mu < \bar{x} + t_{\alpha/2,v}\sqrt{\frac{s^2}{n}}$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}},$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v} \cdot S_p\sqrt{\frac{2}{n}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v} \cdot S_p\sqrt{\frac{2}{n}}; v = 2n-2$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v} \cdot S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v} \cdot S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where Pooled estimate of variance, $S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$ with $v = n_1 + n_2 - 2$,

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v}\sqrt{\frac{1}{n}\left(s_1^2 + s_2^2\right)} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v}\sqrt{\frac{1}{n}\left(s_1^2 + s_2^2\right)} \text{ with } v = 2(n-1),$$

$$\left(\bar{x}_1-\bar{x}_2\right)-t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}<\mu_1-\mu_2<\left(\bar{x}_1-\bar{x}_2\right)+t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \text{ with } v=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}},$$

$$\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2,v}}<\sigma^2<\frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2,v}} \text{ with } v=n-1,$$

$$\frac{s_1^2}{s_2^2}\cdot\frac{1}{f_{\alpha/2}(v_1,v_2)}<\frac{\sigma_1^2}{\sigma_2^2}<\frac{s_1^2}{s_2^2}\cdot f_{\alpha/2}(v_2,v_1) \text{ with } v_1=n_1-1 \text{ and } v_2=n_2-1.$$

Hypothesis Testing :

$$Z=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}},\ T=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{S_p\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \text{ with } v=n_1+n_2-2,$$

$$Z=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}},\ T=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{1}{n}\left(s_1^2+s_2^2\right)}},\ T=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \text{ with}$$

$$v=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}\ ;\ S_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}\ ;\ \chi^2=\frac{(n-1)s^2}{\sigma^2}$$

$$F=\frac{s_1^2}{s_2^2}, \quad \text{with } \frac{1}{f_{\alpha/2}(v_2,v_1)} \quad \text{and} \quad f_{\alpha/2}(v_1,v_2)$$

Simple Linear Regressions :

$$S_{xy}=\sum x_i y_i-\frac{\sum x_i\cdot\sum y_i}{n},\ S_{xx}=\sum x_i^2-\frac{\left(\sum x_i\right)^2}{n},\ S_{yy}=\sum y_i^2-\frac{\left(\sum y_i\right)^2}{n},\ \bar{x}=\frac{\sum x}{n},\ \bar{y}=\frac{\sum y}{n},$$

$$\hat{\beta}_1=\frac{S_{xy}}{S_{xx}},\ \hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x},\ \hat{y}=\hat{\beta}_0+\hat{\beta}_1 x,\ r=\frac{S_{xy}}{\sqrt{S_{xx}\cdot S_{yy}}},\ SSE=S_{yy}-\hat{\beta}_1 S_{xy},\ MSE=\frac{SSE}{n-2},$$

$$T=\frac{\hat{\beta}_1-\beta_1^*}{\sqrt{\frac{MSE}{S_{xx}}}}\sim t_{n-2},\ T=\frac{\hat{\beta}_0-\beta_0^*}{\sqrt{MSE\left(\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right)}}\sim t_{n-2}.$$