UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2010/2011

SUBJECT	:	ENGINEERING MATHEMATICS III
CODE	:	BSM 2913 / BMW 20403
COURSE	:	1BDD / 2 BDD / 3 BDD / BDI / BEE / BEI / BFF /BFI
DATE	:	NOVEMBER / DECEMBER 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF 5 PAGES

Q1 (a) Let G be the "ice cream cone" bounded below by $z=\sqrt{3(x^2+y^2)}$ and above by $x^2+y^2+z^2=4$. Write an iterated integral which gives the volume of G by using spherical coordinates.

(10 marks)

(b) Find the mass of the solid that lies below the paraboloid $z=25-x^2-y^2$ inside the cylinder $x^2+y^2=4$ above the $xy-$plane, and has density function $\rho(x,y,z)=x^2+y^2+6z.$

(10 marks)

Q2 (a) Let $\mathbf{r}(t)=2\cos t\ \mathbf{i}+2\sin t\ \mathbf{j}\ +3t\ \mathbf{k}.$ be a position vector of a particle moving in space.

(i) Find its velocity, speed and acceleration at time $t=\pi$.

(ii) Find the unit tangent vector $\mathbf{T}(t)$, principal unit normal vector $\mathbf{N}(t)$ and curvature κ.

(12 marks)

(b) Evaluate the surface integral

$$\iint_\sigma (x^2z+y^2z)\,dS,$$

where σ is the portion of hemisphere $x^2+y^2+z^2=4,\ z\geq 0.$

(8 marks)

Q3 (a) Locate all relative maxima, relative minima and saddle point, if any for

$$f(x,y)=2x^2-4xy+y^4+2.$$

(12 marks)

(b) Evaluate $\int_C y\,dx+z\,dy+x\,dz,$ where C consists of the line segment C_1 from $(3,4,5)$ followed by the vertical line segment C_2 from $(3,4,5)$ to $(3,4,0)$.

(8 marks)

Q4 (a) Show that the vector field $\mathbf{F}(x,y,z) = 2x(y^2 + z^3)\mathbf{i} + 2yx^2\mathbf{j} + 3x^2z^2e\,\mathbf{k}$ is conservative. Find its scalar potential function $\phi(x,y,z)$ and work done by the force $\mathbf{F}$ in moving a particle from $(-1, 2, 1)$ to $(2, 3, 4)$.

(10 marks)

(b) Use Green's Theorem to evaluate

$$\oint_C \left(3y - e^{\sin x}\right)dx + \left(7x + \sqrt{y^4 + 1}\right)dy$$

where C is the circle $x^2 + y^2 = 9$ oriented counter clockwise.

(10 marks)

Q5 (a) Use Stoke's Theorem to compute the integral

$$\iint_\sigma (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

where $\mathbf{F}(x,y,z) = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$ and σ is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the $xy-$plane.

(10 marks)

(b) Evaluate

$$\iint_\sigma \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F}(x,y,z) = xy\,\mathbf{i} + (y^2 + e^{xz^2})\,\mathbf{j} + \sin(xy)\,\mathbf{k}$ and σ is the surface of the region bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 0$, and $y + z = 2$.

(10 marks)

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Formulae

Polar coordinates: $x = r\cos\theta,\ y = r\sin\theta$ and $x^2 + y^2 = r^2$

$$\iint_R f(x,y)\,dA = \iint_R f(r,\theta)\ r\,dr\,d\theta$$

Cylindrical coordinates: $x = r\cos\theta,\ y = r\sin\theta,\ z = z$ and $x^2 + y^2 = r^2$

$$\iiint_G f(x,y,z)\ dV = \iiint_G f(r,\theta,z)\ r\,dz\,dr\,d\theta$$

Spherical coordinates: $x = \rho\cos\theta\sin\phi,\ y = \rho\sin\theta\sin\phi,\ z = \rho\cos\phi,\ \rho^2 = x^2 + y^2 + z^2,$ $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\ dV = \iiint_G f(\rho,\phi,\theta)\,\rho^2\sin\phi\ d\rho\,d\phi\,d\theta$$

The directional derivatives, $D_u f(x,y) = \left(f_x\,\mathbf{i} + f_y\,\mathbf{j}\right)\cdot\mathbf{u}$; The **gradient** of $\phi = \nabla\phi$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

The **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\dfrac{\partial P}{\partial y} - \dfrac{\partial N}{\partial z}\right)\mathbf{i} - \left(\dfrac{\partial P}{\partial x} - \dfrac{\partial M}{\partial z}\right)\mathbf{j} + \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is smooth curve given by $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$.

The **unit tangent vector**, $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **principal unit normal vector**, $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

Curvature, $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

Green Theorem:

$$\oint_C M\ dx + N\ dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

Gauss Theorem:

$$\iint_S \mathbf{F}\bullet\mathbf{n}\ dS = \iiint_G \nabla\bullet\mathbf{F}\ dV$$

Stokes Theorem:

$$\iint_S (\nabla\times\mathbf{F})\bullet\mathbf{n}\ dS = \int_C \mathbf{F}\bullet d\mathbf{r}$$

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Arc Length of Plane Curve and Space Curve

For a plane curve, $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$ on an interval $[a,b]$, the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\|\, dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt.$$

For a space curve, $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ on an interval $[a,b]$, the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\|\, dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt.$$