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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2010/2011

COURSE NAME	:	MATHEMATICS IV
COURSE CODE	:	BSM 2253
PROGRAMME	:	2BBV / 3BBV / 4BBV
EXAMINATION DATE	:	APRIL / MAY 2011
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B** DO THE CALCULATION IN **4 DECIMAL PLACES**.

THIS EXAMINATION PAPER CONSISTS OF EIGHT (8) PAGES

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PART A

Q1 Given the data of $f(x) = \ln(x)$ in the table below.

x	1.0	1.2	1.4	1.6	1.8
$f(x)$	0.0000	0.1823	0.3365	0.4700	0.5878

(a) Determine the value of $\ln(1.2)$ by using Lagrange polynomial Interpolation. (10 marks)

(b) Evaluate $\ln(1.1)$ by using Newton forward difference method. (5 marks)

(c) Evaluate $\ln(1.7)$ by using Newton backward difference method. (5 marks)

Q2 Given

$$y' = x + xy, \quad y(0) = 1$$

(a) Solve the above first-order differential equation at $x = 0(0.5)2$ by using each methods below.

(i) Second order Taylor's series method. (8 marks)

(ii) Fourth-Order Runge-Kutta Method. (8 marks)

(b) If the exact solution is $y(x) = 2e^{0.5x^2} - 1$, find its errors of each methods below. (4 marks)

PART B

Q3 (a) Obtain the general solution to the linear first order differential equation below.

$$\frac{dr}{d\theta} + r tan\theta = sec\theta$$

(8 marks)

(b) According to Newton's Law of Cooling, if an object at temperature θ°C is immersed in a medium having a constant temperature M°C, then the rate of change if θ°C is proportional to the difference of the temperature $(M - \theta)$°C This gives the ordinary differential equation

$$\frac{d\theta}{dt} = k(M - \theta)$$

(i) Solve the above differential equation for θ. (4 marks)

(ii) A thermometer reading 90°C is placed in a medium having a constant temperature of 65°C. After 5 minutes, the thermometer reads 85°C. What is the reading after 20 minutes?

(8 marks)

Q4 (a) By using variation of parameter method, solve

$$y'' - 2y' + y = \frac{e^x}{1 + x^2} .$$

(11 marks)

(b) By using method of ~~undetermined coefficients~~, solve

$$2x' + x = 3t^2 + 10t .$$

(9 marks)

Q5 (a) Find the approximate value for $\int_1^4 \frac{x}{\sqrt{x+1}} dx$ using trapezoidal rule by taking step size $h = 0.5$.

(6 marks)

(b) Solve the system below by using the Gauss-Seidal iterative method starting with $x^{(0)} = (0 \ 0 \ 0)^T$.

$$4.1x_1 + 9.7x_2 - 2.1x_3 = 20.1$$
$$7.5x_1 - 1.2x_2 + 3.4x_3 = 17.8$$
$$1.2x_1 - 3.1x_2 + 9.3x_3 = 25.9$$

(14 marks)

Q6 (a) Explain briefly the procedure how to solve a system of linear equations $Ax = b$ using $A = LU$ factorization method.

(3 marks)

(b) Given a system of linear equations as below.

$$3x_1 + 2x_2 + 9x_3 = 28$$
$$2x_1 - x_2 + 6x_3 = 14$$
$$5x_1 + 2x_2 - 4x_3 = -13$$

Solve the system by using

(i) Doolittle factorization method. (9 marks)

(ii) Crout factorization method. (8 marks)

~END~

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2010/2011 COURSE : 2 BBV / 3 BBV / 4 BBV
SUBJECT : MATHEMATICS IV SUBJECT CODE : BSM 2253

Formulae

Lagrange Polynomial Interpolation

$$P_n(x) = \sum_{i=0}^{n} L_i(x) f_i$$

For each $k = 0,1,2,3,\ldots,n$ with $L_i(x) = \prod_{j=0,j\neq i}^{n} \frac{(x-x_j)}{(x_i-x_j)}$ and $\sum_{i=0}^{n} L_i(x) = 1$.

Newton Forward Difference Method

$$P_n(x) = f_0 + r\Delta f_0 + \frac{r(r-1)}{2!}\Delta^2 f_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 f_0 + \cdots + \frac{r(r-1)(r-2)\ldots(r-n)}{n!}\Delta^n f_0$$

with $r = \frac{x-x_0}{h}$.

Newton Backward Difference Method

$$P_n(x) = f_n + r\nabla f_n + \frac{r(r+1)}{2!}\nabla^2 f_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 f_n + \cdots + \frac{r(r+1)(r+2)\ldots(r+n)}{n!}\nabla^n f_n$$

with $r = \frac{x-x_n}{h}$.

Taylor Series Method

$$y(x_{i+1}) = y(x_i) + hy'(x_i) + \frac{h^2}{2!}y''(x_i) + \frac{h^3}{3!}y'''(x_i) + \cdots + \frac{h^n}{n!}y^n(x_i)$$

Fourth Order Runge Kutta Method (RK4)

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

with

$$k_1 = hf(x_i, y_i),\ k_2 = hf(x_i + \tfrac{h}{2}, y_i + \tfrac{k_1}{2})$$

$$k_3 = hf(x_i + \tfrac{h}{2}, y_i + \tfrac{k_2}{2}),\ k_4 = hf(x_i + h, y_i + k_3)$$

Trapezoidal Rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f_0 + f_n + 2\sum_{i=1}^{n-1} f_i\right], h = \frac{b-a}{n}$$

Gauss Seidal Iteration Method

$$x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)}}{a_{ii}}, \forall i = 1,2,\dots,n$$

Doolittle Factorization Method

$$A = LU$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

Crout Factorization Method

$$A = LU$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}\begin{pmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$
$P_n(x)\, e^{\alpha x}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\cdot\begin{cases}\cos\beta x\\ \text{or}\\ \sin\beta x\end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)\cos\beta x$ + $x^r(C_n x^n + \cdots + C_1 x + C_0{}^n)\sin\beta x$
$Ce^{\alpha x}\cdot\begin{cases}\cos\beta x\\ \text{or}\\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(p\cos\beta x + q\sin\beta x)$
$P_n(x)e^{\alpha x}\cdot\begin{cases}\cos\beta x\\ \text{or}\\ \sin\beta x\end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x$ + $x^r(C_n x^n + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

Variation of Parameter Method

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$u = -\int \frac{y_2 f(x)}{aW} dx + A$$

$$v = -\int \frac{y_1 f(x)}{aW} dx + B$$

$$y = uy_1 + vy_2$$