# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2010/2011

| | | |
|---|---|---|
| COURSE NAME | : | ENGINEERING STATISTICS |
| COURSE CODE | : | BWM 20502 / BSM 2922 |
| PROGRAMME | : | 2 BFF/ BDD<br>3 BFF/ BDD/ BEE<br>4 BDD/ BEE |
| EXAMINATION DATE | : | APRIL/MAY 2011 |
| DURATION | : | 2 HOURS 30 MINUTES |
| INSTRUCTION | : | ANSWER **ALL** QUESTIONS. |

THIS EXAMINATION PAPER CONSISTS OF SIX PAGES

**Q1** Continuous random variables $X$ has probability density function $f(x)$ as below:

$$f(x)=\begin{cases} kx & 0\le x<1, \\ k & 1\le x<3, \\ k(4-x) & 3\le x\le 4, \\ 0 & otherwise. \end{cases}$$

(a) Find the value of $k$.

(3 marks)

(b) Find $P(X<3)$.

(3 marks)

(c) Find $E(X)$.

(3 marks)

(d) Find $E(2+10X)$.

(3 marks)

(e) Find $Var(X)$.

(5 marks)

(f) Find $Var(4+8X)$.

(3 marks)

**Q2** (a) The number of people arriving for treatment at an emergency room can be modeled by a Poisson process with a mean of five people per hour. What is the probability that

(i) exactly four people arrive during a particular hour.

(ii) at least four people arrive during two hours.

(7 marks)

(b) Previous records show that the scores of XYZ examination are normally distributed with mean 60 and variance 70. Find the probability that a person taking the examination will have

(i) the scores higher than 65.

(ii) the scores between 50 and 70.

(6 marks)

(c) Independent random samples of size $n_1 = 30$ and $n_2 = 50$ are taken from two normal populations having the means $\mu_1 = 78$ and $\mu_2 = 75$ and the variances $\sigma_1^2 = 150$ and $\sigma_2^2 = 200$. Find the probability that the mean of the first sample will exceed the second sample by at least 4.8.

(7 marks)

**Q3** The following summary data on bending strength (lb-in/in) of joints is taken from one of experiments (Refer **Table Q3**).

**Table Q3**: Bending Strength of Two Type of Joints

| Type | Sample Size | Sample Mean | Sample SD |
|---|---|---|---|
| Without side coating | 13 | 80.95 | 9.59 |
| With side coating | 11 | 63.23 | 5.96 |

(a) Construct a 95% confidence interval for the difference between the true average strengths for the two types of joints. Assuming that the random samples were taken from normal population with unequal variances.

(8 marks)

(b) Construct a 95% confidence interval for the variance of a single joint with a side coating.

(6 marks)

(c) Construct a 95% confidence interval for the ratio of the variances of the two populations sampled, $\sigma_A^2 / \sigma_B^2$.

(6 marks)

**Q4** (a) To find out whether the inhabitants of two South Pacific Islands may be regarded as having the same racial ancestry, an anthropologist determines the cephalic indicates of six adult males from each island, getting $\bar{X}_1 = 77.4$ and $\bar{X}_2 = 72.2$ and the corresponding standard deviations $s_1 = 3.3$ and $s_2 = 2.1$. Use 0.01 level of significance to test the null hypothesis $\mu_1 = \mu_2$ against the alternative hypothesis $\mu_1 \neq \mu_2$. Assume that the populations sampled are normal and have unknown and unequal variances.

(8 marks)

(b) The following data (Table **Q4 (b)**) refers to airborne bacteria count both for $n = 8$ carpeted hospital rooms and for $n = 8$ uncarpeted rooms. Does there appear to be a difference in variances of bacteria count between carpeted and uncarpeted rooms? Use $\alpha = 0.05$.

**Table Q4 (b)**: Data of Airborne Bacteria in Both Type of Rooms

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Carpeted** | 11.8 | 8.2 | 7.1 | 13.0 | 10.8 | 10.1 | 14.6 | 14.0 |
| **Uncarpeted** | 12.1 | 8.3 | 3.8 | 7.2 | 12.0 | 11.1 | 10.1 | 13.7 |

(12 marks)

**Q5** The following data represent marks obtained by 10 students in Test 1 and Test 2 (Table **Q5**).

**Table Q5**: Marks Obtained by Students on Test 1 and Test 2

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Test 1, $x$** | 65 | 63 | 76 | 46 | 68 | 72 | 68 | 57 | 36 | 96 |
| **Test 2, $y$** | 68 | 66 | 86 | 48 | 65 | 66 | 71 | 57 | 42 | 87 |

(a) Fit a linear regression model with Test 1 as the explanatory variable and Test 2 as the dependent variable.

(8 marks)

(b) Interpret the meaning of $\beta_1$ value in part (a). Predict the score a student would obtain in Test 2 if he scored 60 marks in Test 1.

(3 marks)

(c) Do the data support the existence of a linear relationship between Test 1 and Test 2? Test the hypothesis against $\beta_1 \neq 0$ using $\alpha = 0.05$.

(6 marks)

(d) Find the Pearson correlation coefficient value. How the value described the relationship between Test 1 and Test 2?

(3 marks)

**FINAL EXAMINATION**

SEMESTER / SESSION: SEM II / 2010/2011 | COURSE: 2 BFF/ BDD, 3 BFF/ BDD/ BEE, 4 BEE/ BDD

SUBJECT : ENGINEERING STATISTICS | CODE: BWM 20502 / BSM 2922

**Formulae**

Random variables:

$$\sum_{i=-\infty}^{\infty} P(x_i) = 1, \quad E(X) = \sum_{\forall x} x \cdot P(x), \quad E(X^2) = \sum_{\forall x} x^2 \cdot P(x), \quad \int_{-\infty}^{\infty} f(x)\,dx = 1,$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot P(x)\,dx, \quad E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot P(x)\,dx, \quad Var(X) = E(X^2) - [E(X)]^2.$$

Special Probability Distributions :

$$P(x=r) = {}^nC_r \cdot p^r \cdot q^{n-r},\ r = 0, 1, \ldots, n,\ X \sim B(n, p),\ P(X=r) = \frac{e^{-\mu} \cdot \mu^r}{r!},\ r = 0, 1, \ldots, \infty,$$

$$X \sim P_0(\mu),\ Z = \frac{X-\mu}{\sigma},\ Z \sim N(0,1),\ X \sim N(\mu, \sigma^2).$$

Sampling Distributions :

$$\bar{X} \sim N(\mu, \sigma^2/n),\ Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1),\ T = \frac{\bar{x}-\mu}{s/\sqrt{n}},\ \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right).$$

Estimations :

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2,\ \left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}},$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v} \cdot S_p\sqrt{\frac{2}{n}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v} \cdot S_p\sqrt{\frac{2}{n}};\ v = 2n-2$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v} \cdot S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v} \cdot S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where Pooled estimate of variance, $S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$ with $v = n_1 + n_2 - 2$,

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v}\sqrt{\frac{1}{n}\left(s_1^2 + s_2^2\right)} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v}\sqrt{\frac{1}{n}\left(s_1^2 + s_2^2\right)} \text{ with } v = 2(n-1),$$

$$\left(\bar{x}_1-\bar{x}_2\right)-t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}<\mu_1-\mu_2<\left(\bar{x}_1-\bar{x}_2\right)+t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \text{ with } v=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}},$$

$$\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2,v}}<\sigma^2<\frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2,v}} \text{ with } v=n-1,$$

$$\frac{s_1^2}{s_2^2}\cdot\frac{1}{f_{\alpha/2}(v_1,v_2)}<\frac{\sigma_1^2}{\sigma_2^2}<\frac{s_1^2}{s_2^2}\cdot f_{\alpha/2}(v_2,v_1) \text{ with } v_1=n_1-1 \text{ and } v_2=n_2-1.$$

Hypothesis Testing :

$$Z=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}},\; T=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{S_p\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \text{ with } v=n_1+n_2-2,$$

$$Z=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}},\; T=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{1}{n}\left(s_1^2+s_2^2\right)}},\; T=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \text{ with}$$

$$v=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}\;;\; S_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}\;;\; \chi^2=\frac{(n-1)s^2}{\sigma^2}$$

$$F=\frac{s_1^2}{s_2^2}, \quad \text{with } \frac{1}{f_{\alpha/2}(v_2,v_1)} \quad \text{and} \quad f_{\alpha/2}(v_1,v_2)$$

Simple Linear Regressions :

$$S_{xy}=\sum x_i y_i-\frac{\sum x_i\cdot\sum y_i}{n},\; S_{xx}=\sum x_i^2-\frac{\left(\sum x_i\right)^2}{n},\; S_{yy}=\sum y_i^2-\frac{\left(\sum y_i\right)^2}{n},\; \bar{x}=\frac{\sum x}{n},\; \bar{y}=\frac{\sum y}{n},$$

$$\hat{\beta}_1=\frac{S_{xy}}{S_{xx}},\; \hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x},\; \hat{y}=\hat{\beta}_0+\hat{\beta}_1 x,\; r=\frac{S_{xy}}{\sqrt{S_{xx}\cdot S_{yy}}},\; SSE=S_{yy}-\hat{\beta}_1 S_{xy},\; MSE=\frac{SSE}{n-2},$$

$$T=\frac{\hat{\beta}_1-\beta_1^*}{\sqrt{\frac{MSE}{S_{xx}}}}\sim t_{n-2},\; T=\frac{\hat{\beta}_0-\beta_0^*}{\sqrt{MSE\left(\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right)}}\sim t_{n-2}.$$