UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2010/2011

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	BWM 10103 / BSM 1913
PROGRAMME	:	1 BFF / BEF / BEB / BEH / BDD 2 BFF 3 BFF / BEE
EXAMINATION DATE	:	APRIL/MAY 2011
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FOUR (4)** QUESTIONS ONLY

THIS EXAMINATION PAPER CONSISTS OF SIX (6) PAGES

Q1 (a) Find $\frac{dy}{dx}$ for the following inverse functions.

(i) $y(x)=(x^2+1)\csc^{-1}4x.$

(ii) $y(x)=\cosh^{-1}(\sec x).$

(6 marks)

(b) Evaluate

(i) $\int \frac{e^{2x}}{\sqrt{e^{4x}+16}}\,dx.$

(ii) $\int_0^{1/2} 2\cos^{-1}x\,dx.$

(13 marks)

(c) Find the surface area generated when curve of $g(y)=\sqrt{9-y^2}$ is rotated 360° from $y=-2$ to $y=2$ about the y-axis.

(6 marks)

Q2 (a) Find the radius and interval of convergence for the given power series

$$\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n}.$$

(10 marks)

(b) Given $f(x)=e^{5x}$. Obtain

(i) Maclaurin series expansion of $f(x)$.

(ii) Taylor series expansion of $f(x)$ at the point $x=1$.

(10 marks)

(c) For the power series given by

$$\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}.$$

Evaluate $\int_0^1 \cos(x^3)\,dx$ up to 3 terms only.

(5 marks)

Q3 (a) Evaluate the following limits if exist.

(i) $\lim_{x\to+\infty} x^2 e^{-3x}$.

(ii) $\lim_{x\to 0} \dfrac{\tan x - x}{x - \sin x}$.

(iii) $\lim_{x\to 1} \dfrac{1}{\ln x} - \dfrac{1}{x-1}$.

(iv) $\lim_{x\to 1} \dfrac{\sin\left(1-\sqrt{x}\right)}{x-1}$.

(15 marks)

(b) Given the function

$$f(x) = \begin{cases} 2x^3 + x + 7, & x \le -1, \\ m(x+1) + k, & -1 < x \le 2, \\ x^2 + 5, & x > 2. \end{cases}$$

Determine the value of m and k such that the function $f(x)$ continuous everywhere.

(7 marks)

(c) Find the values of x so that the function below is discontinuous.

$$f(x) = \frac{x^2 - 16}{x^2 - 5x + 4}.$$

(3 marks)

Q4 (a) A rectangular water tank (**Figure Q4 (a)**) is being filled at the constant rate of 30 liters/second. The base of the tank has dimensions $w = 1$ meter and $L = 2$ meters. What is the rate of change of the height of water in the tank?
(Give the answer in cm/second)

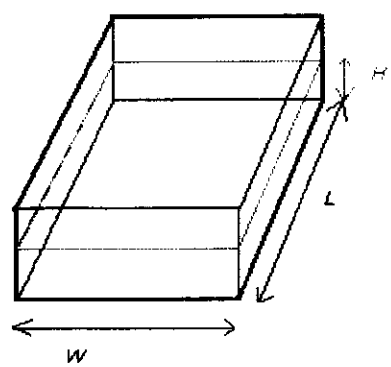

Figure Q4(a)

(4 marks)

(b) Find $\frac{dy}{dx}$ for $y^2 + x^3 - xy + \cos y = 0$ by using implicit differentiation.

(6 marks)

(c) Sketch the rational function

$$f(x) = \frac{x}{x^2 - 4}.$$

Show all the asymptote(s), intersection point(s), extremum and inflection point(s) (if any) in your graph.

(15 marks)

Q5 (a) Evaluate

(i) $\int_0^1 x^3 e^{2x}\, dx.$

(ii) $\int \sin^4 3x \cos^5 3x\, dx.$

(iii) $\int \frac{\sqrt{3+\sqrt{x}}}{\sqrt{x}}\, dx.$ [Use substitution method using $u = 3 + \sqrt{x}$]

(15 marks)

(b) Find $\int \sqrt{1+\cot^2\theta}\, d\theta$ given that $\int \csc\theta\, d\theta = \ln|\csc\theta - \cot\theta| + C.$

(3 marks)

(c) Show that $\int \frac{\sin x}{\sin x + \tan x}\, dx = 2\tan^{-1}\left(\tan\frac{x}{2}\right) - \tan\frac{x}{2} + C$ by using the substitution $t = \tan\frac{x}{2}.$

(7 marks)

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x \, dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C, \ |x|<1$$

$$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C, \ |x|<1$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C, \ |x|>1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}} dx = \csc^{-1} x + C, \ |x|>1$$

$$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C, \ |x|>1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}} dx = \operatorname{sech}^{-1} |x| + C, \ 0<x<1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}} dx = \operatorname{csch}^{-1} |x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2} dx = \begin{cases} \tanh^{-1} x + C, & |x|<1 \\ \coth^{-1} x + C, & |x|>1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$