UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2010/2011

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BWM10203 / BSM1923
PROGRAMME	:	1 BFF/ BDD 2 BFF/ BDD 3 BFF/ BDD 4 BFF/ BDD
EXAMINATION DATE	:	APRIL / MAY 2011
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF SIX (6) PAGES

PART A

Q1 A periodic function is defined by

$$f(x)=\begin{cases} 0, & -\pi < x < -\dfrac{\pi}{2}, \\ 4, & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}, \\ 0, & \dfrac{\pi}{2} < x < \pi, \end{cases}$$

$$f(x)=f(x+2\pi).$$

(a) Sketch the graph of the function over $-2\pi < x < 2\pi$.

(4 marks)

(b) Determine whether the function is even, odd or neither.

(1 marks)

(c) Show that the Fourier series of the function $f(x)$ is

$$2+\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(n\pi/2\right)}{n}\cos nx .$$

(15 marks)

Q2 An insulated uniform metal bar, 2 units long, has the temperature of its ends maintained at 0°C and at $t=0$ the temperature distribution $f(x)$ along the bar is defined by $f(x)=2x^2-4x$. Given a heat conduction equation

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{c^2}\frac{\partial u}{\partial t}$$

with $c^2=9$.

(a) By separation of variables, the solution of heat equation above can be written as $u(x,t)=X(x)T(t)$. By implementing this method and applying the boundary conditions, show that

$$u(x,t)=\sum_{n=1}^{\infty}A_n\sin\left(\frac{n\pi x}{2}\right)e^{-9\left(\frac{n\pi}{2}\right)^2 t},$$

where A_n is an arbitrary constant.

(14 marks)

(b) Then, apply initial condition to find A_n.

(6 marks)

PART B

Q3 (a) Solve the following linear differential equation

$$(\cos x)\frac{dy}{dx}+(\sin x)y=2\cos^3 x\sin x-1$$

when $y\left(\frac{\pi}{4}\right)=3\sqrt{2}$.

(11 marks)

(b) Given a differential equation

$$\left[\left(\frac{2x}{x^2+1}\right)y-2x\right]dx+\left[\ln\left(x^2+1\right)-2\right]dy=0.$$

(i) Show that the equation is exact.

(ii) Solve the differential equation for $y(x)$.

(9 marks)

Q4 (a) Solve $y''+9y=\sec 3x$ by using variation of parameters method.

(12 marks)

(b) The equation of motion is governed by

$$\ddot{x}+\frac{a}{M}\dot{x}+\frac{k}{M}x=\frac{F(t)}{M}$$

where M is a mass, a is resistance, k is a spring constant and $F(t)$ is an external force. A steel ball has a mass of 2 kg is suspended from a spring with a known spring constant of 32 N/m.

(i) Assuming no air resistance and no external force, show that an expression for the position of the ball at any time t is

$$x(t)=A\cos 4t+B\sin 4t.$$

(ii) Hence, find A and B if $x(0)=0$ and $\dot{x}(0)=1$.

(8 marks)

Q5 (a) Show that $\mathcal{L}\{\, t\cos 4t \,\} = \dfrac{s^2-16}{\left(s^2+16\right)^2}$.

(7 marks)

(b) From **Q5**(a), find $\mathcal{L}\left\{\, t e^{-3t}\cos 4t \,\right\}$.

(3 marks)

(c) Consider the function

$$g(t) = \begin{cases} 0, & 0 \le t < \pi, \\ -\sin t\,, & t \ge \pi. \end{cases}$$

(i) Write the function $g(t)$ in the form of Heaviside / unit step function.

(ii) Then, find the Laplace transform of $g(t)$.
[Hint: $\sin(A+B) = \sin A\cos B + \cos A\sin B$]

(10 marks)

Q6 (a) Find the inverse Laplace transform of the following functions.

(i) $Y(s) = \dfrac{e^{-\pi s}}{(s^2+1)}$.

(ii) $Y(s) = \dfrac{s}{s^2-6s+18}$.

(7 marks)

(b) Given the second order linear differential equation

$$y'' + 5y' + 6y = e^{-t}$$

with the boundary conditions, $y(0)=0$ and $y'(0)=0$.

(i) Show that by using Laplace transform, the differential equation above becomes

$$Y(s) = \frac{1}{(s+1)(s^2+5s+6)}.$$

(ii) Hence, solve $\mathcal{L}^{-1}\{Y(s)\}$.

(13 marks)

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$
$P_n(x)\, e^{\alpha x}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x) \cdot \begin{cases} \cos\beta x \\ \sin\beta x \end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x} \cdot \begin{cases} \cos\beta x \\ \sin\beta x \end{cases}$	$x^r e^{\alpha x}(p\cos\beta x + q\sin\beta x)$
$P_n(x)e^{\alpha x} \cdot \begin{cases} \cos\beta x \\ \sin\beta x \end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W}dx, \quad u_2 = \int \frac{y_1 f(x)}{W}dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt, \quad s > 0.$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$