UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2010/2011

SUBJECT	:	ENGINEERING MATHEMATICS I
CODE	:	BWM 10103 / BSM 1913
PROGRAMME	:	1 BFF / 2 BFF / 3 BFF / 4 BFF 1 BEE / 2 BEE / 3 BEE / 4 BEE 1 BDD / 2 BDD
DATE	:	NOVEMBER / DECEMBER 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS.

THIS EXAMINATION PAPER CONSISTS OF 5 PAGES

Q1 (a) Evaluate the following limits (if exist).

(i) $\lim\limits_{x \to 9} \dfrac{3-x}{\sqrt{3x}-3}$

(ii) $\lim\limits_{x \to 0} \dfrac{\sinh x - x^2}{3\cosh x - 3}$

(iii) $\lim\limits_{x \to 0} \left[\left(\dfrac{3}{x} \right) \left(\dfrac{1}{5+x} - \dfrac{1}{5-x} \right) \right]$

(iv) $\lim\limits_{x \to \pi} \left(\dfrac{1}{x-\pi} \right) \sin 2x$

(13 marks)

(b) Show that

$$f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases}$$

is a continuous function for all $x \in R$.

(7 marks)

Q2 (a) Given a rational function

$$f(x) = \frac{x}{x^2-1}.$$

(i) Show that $f(x)$ has a vertical asymptotes at $x = -1$, 1 and horizontal asymptote at x-axis.

(ii) Find critical point(s) (if any) and inflection point(s) for $f(x)$.

(iii) Show that x-intercept and y-intercept are represented by origin point.

(iv) Sketch the graph for $f(x)$ based on information obtained in (i), (ii) and (iii).

(12 marks)

(b) Each edge of a variable cube is increasing at a rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 12 cm long?

(4 marks)

(c) Show that

$$2y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 - 2y^4 = \sec^4 x - \sec^2 x,$$

if $y = \sec x$.

(4 marks)

Q3 (a) Use logarithmic differentiation to find $\frac{dy}{dx}$ for $y=\left(\sin^{-1}x\right)^{\sqrt{x}}$.

(5 marks)

(b) Evaluate

(i) $\int_0^{\pi/2} \sin 2x \cos 4x \, dx$.

(ii) $\int \frac{1}{x\left[1+(\ln x)^2\right]} dx$.

(7 marks)

(c) Show that

$$\int x \operatorname{sech}^{-1} x \, dx = \frac{x^2}{2}\operatorname{sech}^{-1} x - \frac{1}{2}\sqrt{1-x^2} + C.$$

(8 marks)

Q4 (a) Find a curvature, κ of $x=t^4-t^3$ and $y=t^3-t^2$ at $t=1$.

(5 marks)

(b) Find the arc length of

$$x = 8\cos t + 8t\sin t,\ y = 8\sin t - 8t\cos t;\ 0 \le t \le 1/2.$$

(8 marks)

(c) Find the area of the surface when the arc of $g(y)=2\sqrt{1-y}$ from $y=-1$ to $y=0$ revolved about the y-axis.

(7 marks)

Q5 (a) Consider the power series

$$\sum_{n=0}^{\infty}\frac{(x-8)^n}{n+1}.$$

Find the radius and interval of convergence of the given power series.

(12 marks)

(b) Show that the Maclaurin series for $f(x)=e^x$ is

$$1+x+\frac{x^2}{2}+\cdots$$

Then, evaluate $\int_0^1 e^{x^3} dx$.

(8 marks)

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \csc^2 x\, dx = -\cot x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \csc x \cot x\, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x\, dx = \sinh x + C$$

$$\int \sinh x\, dx = \cosh x + C$$

$$\int \text{sech}^2 x\, dx = \tanh x + C$$

$$\int \text{csch}^2 x\, dx = -\coth x + C$$

$$\int \text{sech}\, x \tanh x\, dx = -\text{sech}\, x + C$$

$$\int \text{csch}\, x \coth x\, dx = -\text{csch}\, x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \; |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \; |x| < 1$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \; |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \; |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \; |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \text{sech}^{-1} |x| + C, \; 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \text{csch}^{-1} |x| + C, \; x \neq 0$$

$$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	***Trigonometry***	***Hyperbolic***
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

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Formulae

TRIGONOMETRIC SUBSTITUTION

$t = \tan \frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$
$\tan x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{2dt}{1+t^2}$	$\tan 2x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x + \sin^2 x = 1$$
$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$
$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$2\sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$$
$$2\sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$$
$$2\cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$
$$1 - \tanh^2 x = \text{sech}^2 x$$
$$\coth^2 x - 1 = \text{csch}^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$
$$\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$S = 2\pi\int_{x_1}^{x_2} f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\,dx$$

$$S = 2\pi\int_{y_1}^{y_2} g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\,dy$$