UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2010/2011

COURSE	:	ENGINEERING MATHEMATICS II
CODE	:	BSM1923 / BWM10203
PROGRAMME	:	1 BFF / BDD, 2 BDD, 3 BFF/BDD, 4 BFF/ BDD
DATE	:	NOVEMBER / DECEMBER 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS.

THIS EXAMINATION PAPER CONSISTS OF 6 PAGES

Q1 A periodic function $f(x)$ is defined by

$$f(x)=\begin{cases}-\pi, & -\pi<x<0,\\ x, & 0<x<\pi,\end{cases}$$
$$= f(x+2\pi).$$

(a) Sketch the graph of the function over the interval $-3\pi<x<3\pi$.

(3 marks)

(b) Find the value of coefficient a_0.

(3 marks)

(c) Show that for $n=1,2,3,\ldots$

$$a_n=\frac{1}{n^2\pi}(\cos n\pi-1) \text{ and } b_n=\frac{1}{n}(1-2\cos n\pi).$$

(11 marks)

(d) Hence, determine a Fourier series expansion for $f(x)$.

(3 marks)

Q2 (a) Show that the function $u(x,t)=(x-ct)^3+(x+ct)^3$ is a solution of $\dfrac{\partial^2 u}{\partial t^2}=c^2\dfrac{\partial^2 u}{\partial x^2}$.

(6 marks)

(b) A stretched string of length 4cm is set oscillating. The initial state is given by a function, $f(x)=x(4-x)$. The string has been released with zero initial velocity. By applying the wave equation $u_{xx}=u_{tt}$, the subsequent displacement of the string is

$$u(x,t)=\sum_{n=1}^{\infty}\left[A_n\cos\left(\frac{n\pi}{4}\right)t+B_n\sin\left(\frac{n\pi}{4}\right)t\right]\sin\left(\frac{n\pi}{4}\right)x.$$

(i) List the initial and boundary conditions.
(ii) Use the Fourier series method to find the value of A_n and B_n.

$$\left[\text{Hint}: A_n=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}\right)x\,dx,\ B_n=\frac{2}{n\pi c}\int_0^L g(x)\sin\left(\frac{n\pi}{L}\right)x\,dx.\right]$$

(14 marks)

Q3 (a) Given the first order ordinary differential equation

$$\left(1-\cos^2 x\right)dy + y\sin 2x dx = 0, \quad y(0) = -1.$$

(i) Show that the given equation is exact.
(ii) Hence, find the general solution of the exact equation.

(10 marks)

(b) The equation of motion is governed by

$$x'' + \frac{a}{M}x' + \frac{k}{M}x = \frac{F(t)}{M}$$

where M is a mass and k is a spring constant. Suppose that a mass of 2 kg is suspended from a spring with a known spring constant of 4 N/m and allowed to come to rest. It is then set in motion by giving it an initial velocity of 2 m/s. Find the position of the mass at any time if the magnitude of the external force, $F(t) = 2t$ and the air resistance, $a = 6$.

(10 marks)

Q4 (a) Find the Laplace transform for each of the following function.

(i) $f(t) = e^{-t}\sin 5t\cos 5t$.
(ii) $f(t) = t^2\sinh 2t$.
(iii) $f(t) = 3(t-1)H(t-3)$.

(11 marks)

(b) Consider the periodic function

$$g(t) = \begin{cases} e^{-2t}, & 0 \le t < 2, \\ -2, & 2 \le t < 4, \end{cases}$$
$$= g(t+4).$$

Find the Laplace transform of $g(t)$.

(9 marks)

Q5 (a) Find

(i) $\mathcal{L}^{-1}\left\{\dfrac{3e^{-t}}{s^4}\right\}$.

(ii) $\mathcal{L}^{-1}\left\{\dfrac{s+2}{2s^2-12s+13}\right\}$.

(9 marks)

(b) Solve the differential equation

$$y''+4y'+4y=1+\delta(t-2)$$

when $y(0)=0$ and $y'(0)=0$.

(11 marks)

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$
$P_n(x)\, e^{\alpha x}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x) \cdot \begin{cases} \cos\beta x \\ \sin\beta x \end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + \cdots + C_1 x + C_0{}^n)\sin\beta x$
$Ce^{\alpha x} \cdot \begin{cases} \cos\beta x \\ \sin\beta x \end{cases}$	$x^r e^{\alpha x}(p\cos\beta x + q\sin\beta x)$
$P_n(x)e^{\alpha x} \cdot \begin{cases} \cos\beta x \\ \sin\beta x \end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W}dx, \quad u_2 = \int \frac{y_1 f(x)}{W}dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt, \quad s > 0.$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$