UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2010/2011

COURSE	:	MATHEMATICS III
CODE	:	BSM 2223
PROGRAMME	:	2 BBV/ 3 BBV
DATE	:	NOVEMBER/DECEMBER 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF 7 PAGES

Q1 (a) The following probabilities are given:

$P(A_1) = 0.45$	$P(A_2) = 0.35$	$P(A_3) = 0.20$
$P(H \setminus A_1) = 0.10$	$P(H \setminus A_2) = 0.12$	$P(H \setminus A_3) = 0.15$

Find:

(i) $P(A_1 \setminus H)$ (2 marks)

(ii) $P(A_2 \mid H)$ (2 marks)

(b) Particles are a major component of air pollution in many areas. It is of interest to study the size of contaminating particles. Let X represent the diameter, in micrometers, of a randomly chosen particle. Assume that in certain area, the probability density function of X is inversely proportional to the volume of the particle, that is, assume that

$$f(x) = \begin{cases} cx^3 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

where c is constant.

(i) Find the value of c so that $f(x)$ is probability density function. (3 marks)

(ii) Find the mean of the particle diameter. (4 marks)

(iii) Find the variance of the particle diameter. (4 marks)

(iv) Find the cumulative distribution function of the particle diameter. (5 marks)

Q2. (a) The number of message received by computer bulletin board is a Poisson random variable with a mean rate of 8 messages per hour.

(i) What is the probability that five messages are received in a given hour? (3 marks)

(ii) Find the mean and variance number of message received. (2 marks)

(iii) What is the probability that fewer than three messages are received in one and half hour? (3 marks)

(b) The fill volume of cans filled by a certain machine is normally distributed with mean 12.05 ml and standard deviation 0.03 ml.

(i) What is the probability that the cans contain less than 12.00 ml?
(3 marks)

(ii) What is the probability that the cans contain between 12.00 and 12.11 ml?
(4 marks)

(iii) If **nine** cans were randomly selected, what is the probability that the average cans contain is greater than 12.025 ml?
(5 marks)

Q3 (a) One step in the manufacture of a certain metal clamp involves the drilling of four holes. In a sample of 25 clamps, the average time needed to complete this step was 70 seconds and the standard deviation was 10 seconds.

(i) Find a 95% confidence interval for the mean time needed to complete the step.
(4 marks)

(ii) Find a 95% confidence interval for the standard deviation time needed to complete the step.
(4 marks)

(b) Two weights, each labeled as weighting 100 g, are each weighed several times on the same scale. The results in units of μg above 100 g, are as follows:

First weight :	53	88	89	62	39	66
Second weight:	23	39	28	52	49	

(i) At 5% level of significance, can you conclude that the variances from two weights are same?
(6 marks)

(ii) At 5% level of significance, can you conclude that the mean weights are differ?
(6 marks)

Q4 Inertial weights (in tons) and fuel economy (in mi/gal) were measured for a sample of seven cars. The results are presented in the **Table Q4**.

Table Q4: The inertial weights and fuel economy

Weight	8.00	24.50	27.00	14.5	28.50	12.75	21.25
Mileage	7.69	4.97	4.56	6.49	4.34	6.24	4.45

(a) Construct a scatterplot of mileage (y) versus weight (x). Verify that a linear model is appropriate.

(2 marks)

(b) Compute the least square line for predicting mileage from weight and interpret your results.

(6 marks)

(c) Predict the mileage for cars with a weight of 15 tons.

(2 marks)

(d) Compute the coefficient of determination and interpret your results.

(3 marks)

(e) Test the $H_0 : \beta_1 = 0$ versus $H_1 : \beta \neq 0$ at 5% level of significance.

(7 marks)

Q5 (a) The following data represent the number of defective items for each Month (Refer **Table Q5(a)**).

Table Q5(a): Number of defective items form two machines

Machine\ Week	1	2	3	4	5	6	7	8	9	10	11	12
M1	42	47	66	70	67	45	56	60	74	47	61	52
M2	41	49	61	68	69	42	56	57	68	47	59	51

Machine\ Week	13	14	15	16
M1	57	69	68	49
M2	54	63	72	46

Test the hypothesis H_0 that there is no difference between machines against the alternative hypothesis, H_1 that machine 1 is better than machine 2 at the 5% level of significance.

(5 marks)

(b) The following data represent the number of minutes that a customer had to wait on 10 visits before being entertained by the customer relationship officer (Refer **Table Q5(b)**).

Table Q5(b): The number of minutes that a ten customer had to wait

28	22	30	28	29
25	15	18	24	19

Use the sign-rank test at the 0.05 level of significance to test the officer claim that the median waiting time is not more than 20 minutes.

(5 marks)

(c) A company wishes to purchase one of 3 different machines, *M1*, *M2* or *M3*. In an experiment designed whether the performance difference between the machines, three experienced operators each work on the machines for equal times. The number of unit produced by each machines is shown in **Table Q5(c).**

Table Q5(c): The number of unit produced by each machines

Machine *M1*	53	68	64	70	65
Machine *M1*	48	53	72	63	55
Machine *M1*	53	42	68	72	77

Test the hypothesis that there is no difference between the machines at the 0.05 level of significance.

(10 marks)

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Formulae

Random variables:

$$\sum_{i=-\infty}^{\infty} P(x_i) = 1, \quad E(X) = \sum_{\forall x} x \cdot P(x), \quad E(X^2) = \sum_{\forall x} x^2 \cdot P(x), \quad \int_{-\infty}^{\infty} f(x)\, dx = 1,$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot P(x)\, dx, \quad E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot P(x)\, dx,$$

$$Var(X) = E(X^2) - [E(X)]^2.$$

Special Probability Distributions :

$$P(x = r) = {}^nC_r \cdot p^r \cdot q^{n-r},\ r = 0, 1, \ldots, n,\ X \sim B(n, p),$$

$$P(X = r) = \frac{e^{-\mu} \cdot \mu^r}{r!},\ r = 0, 1, \ldots, \infty,\ X \sim P_0(\mu),\ Z = \frac{X - \mu}{\sigma},\ Z \sim N(0, 1),$$

$$X \sim N(\mu, \sigma^2).$$

Sampling Distributions :

$$\bar{X} \sim N(\mu, \sigma^2/n),\ Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1),\ T = \frac{\bar{x} - \mu}{s/\sqrt{n}},\ \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right).$$

Estimations :

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2,\ \left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}},$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v} \cdot S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v} \cdot S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where Pooled estimate of variance, $S_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ with $v = n_1 + n_2 - 2$,

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v}\sqrt{\frac{1}{n}\left(s_1^2 + s_2^2\right)} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v}\sqrt{\frac{1}{n}\left(s_1^2 + s_2^2\right)} \text{ with } v = 2(n-1),$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \text{ with}$$

$$v = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(\dfrac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \dfrac{\left(\dfrac{s_2^2}{n_2}\right)^2}{n_2 - 1}}, \quad \frac{(n-1) \cdot s^2}{\chi^2_{\alpha/2,v}} < \sigma^2 < \frac{(n-1) \cdot s^2}{\chi^2_{1-\alpha/2,v}} \text{ with } v = n - 1,$$

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$$\frac{s_1^2}{s_2^2}\cdot\frac{1}{f_{\alpha/2}(v_1,v_2)}<\frac{\sigma_1^2}{\sigma_2^2}<\frac{s_1^2}{s_2^2}\cdot f_{\alpha/2}(v_2,v_1) \text{ with } v_1=n_1-1 \text{ and } v_2=n_2-1.$$

Hypothesis Testing:

$$Z=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}},\ T=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{S_p\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \text{ with } v=n_1+n_2-2,$$

$$Z=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}},\ T=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{1}{n}\left(s_1^2+s_2^2\right)}},\ S_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$$

$$T=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \text{ with } v=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}.\ ;\ \chi^2=\frac{(n-1)s^2}{\sigma^2}$$

Simple Linear Regressions :

$$S_{xy}=\sum x_iy_i-\frac{\sum x_i\cdot\sum y_i}{n},\ S_{xx}=\sum x_i^{\,2}-\frac{\left(\sum x_i\right)^2}{n},\ S_{yy}=\sum y_i^{\,2}-\frac{\left(\sum y_i\right)^2}{n},\ \bar{x}=\frac{\sum x}{n},$$

$$\bar{y}=\frac{\sum y}{n},\ \hat{\beta}_1=\frac{S_{xy}}{S_{xx}},\ \hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x},\ \hat{y}=\hat{\beta}_0+\hat{\beta}_1x,\ r=\frac{S_{xy}}{\sqrt{S_{xx}\cdot S_{yy}}},$$

$$SSE=S_{yy}-\hat{\beta}_1S_{xy},\ MSE=\frac{SSE}{n-2},\ T=\frac{\hat{\beta}_1-\beta_1^*}{\sqrt{\frac{MSE}{S_{xx}}}}\sim t_{n-2},\ T=\frac{\hat{\beta}_0-\beta_0^*}{\sqrt{MSE\left(\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right)}}\sim t_{n-2}.$$

$$w_1+w_2=\frac{(n_1+n_2)(n_1+n_2+1)}{2};\ u_1=w_1-\frac{n_1(n_1+1)}{2};\ u_2=w_2-\frac{n_1(n_1+1)}{2}$$

$$h=\frac{12}{n(n+1)}\sum_{i=1}^{k}\frac{r_i^2}{n_i}-3(n+1);\ r_s=1-6\sum\frac{d_i^2}{n(n^2-1)}$$