UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2009/2010

SUBJECT	:	ENGINEERING MATHEMATICS IV
CODE	:	BSM 3913
COURSE	:	3BDD / BEE / BFF / BDI
DATE	:	NOVEMBER 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B** ALL CALCULATIONS MUST BE IN THREE DECIMAL PLACES

THIS EXAMINATION PAPER CONSISTS OF 8 PAGES

PART A

Q1 (a) Given the heat equation,

$$u_t = u_{xx}, \quad 0 < x < 1, \; t > 0,$$

with conditions, $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin \pi x$.

By using explicit finite-difference method, find the approximate solution to the heat equation for $x = 0$ to 1 and $t \le 0.04$ only. Take $\Delta x = 0.2, \; \Delta t = 0.02$.

(7 marks)

(b) Given the wave equation,

$$u_{tt} = u_{xx}, \quad 0 < x < 1, \quad 0 < t < 0.3,$$

with the boundary conditions,

$$u(0,t) = u(1,t) = 0, \quad 0 \le t \le 0.3,$$

and the initial conditions,

$$u(x,0) = \sin \pi x, \quad u_t(x,0) = 0, \; \text{for } 0 \le x \le 1.$$

By taking $h = \Delta x = 0.2$ and $k = \Delta t = 0.1$, solve for u using finite-difference method.

(13 marks)

Q2 The steady state temperature distribution of heated rod follows the one-dimensional form of Poisson's equation

$$\frac{d^2T}{dx^2} = -Q(x).$$

Solve the above equation for a 6cm rod with boundary conditions of $T(0,t) = 45^0$ and $T(6,t) = 345^0$ and a uniform heat source $Q(x) = 30$ with 2 equal-size elements of length 3cm by using finite-element method with linear approximation.

(20 marks)

PART B

Q3 (a) The following simultaneous nonlinear equations,

$$f(x) = 2x^3 + 5,$$

$$g(x) = 11 - 2^x$$

are illustrated in **Figure Q3**.

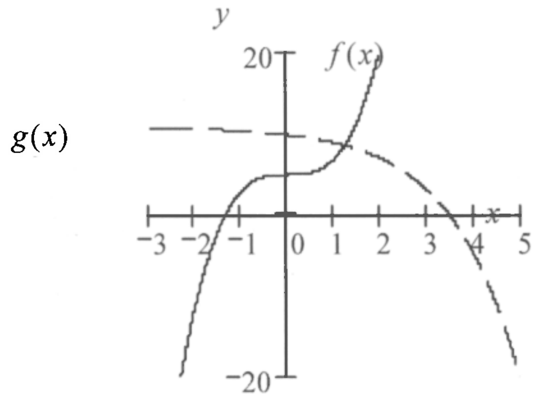

Figure Q3

(i) By Intermediate Value Theorem, estimate the interval of (a, b) consists of x-value which is the intersection of the simultaneous nonlinear equations above.

(ii) Hence, by using Secant method, find the x-value in (a)(i).

(9 marks)

(b) UTHM Publisher publishes "Numerical Method" book in three different bindings: paperback, hardcover and deluxe. Each paperback book needs 1 minute of sewing, 2 minutes of gluing and 4 minutes of packing. Each hardcover book requires 2 minutes of sewing, 6 minutes of gluing and 2 minutes of packing. While each deluxe book takes 4 minutes of sewing, 3 minutes of gluing and 1 minute of packing.

Given that the sewing machine is available 6 hours per day, the gluing machine is available 11 hours per day and the packing machine is available 10 hours per day.

(i) Assume that x representing paperback book, y representing hardcover book and z representing deluxe book. Based on the information above, form the system of linear equations.

(ii) Based on the system of linear equations in **Q3**(b)(i), you are required to determine how many books of each type can be published per day by using Gauss-Seidel Iteration method.

(11 marks)

Q4 (a) The **Table Q4(a)** below gives the values of distance traveled by a motorcycle at various times from a junction.

Table Q4(a)

Time, t (minute)	3	5	7	9	11
Distance traveled, x (km)	4.6	8.03	11.97	16.89	19.9

(i) Find the velocity of the motorcycle at $t = 5$ minutes by 3-point forward difference formula with $h = 2$ minutes.

(ii) By taking $h = 2$ minutes, estimate the acceleration of the motorcycle at $t = 7$ minutes by 5-point difference formula.

[Hint: acceleration, $a = \dfrac{dv}{dt}$ and velocity, $v = \dfrac{dx}{dt}$].

(6 marks)

(b) Construct a Natural Cubic Spline that interpolates to the following data, shown in **Table Q4(b).**

Table Q4(b)

x	1	2	3	4
$f(x)$	1	1	0	-1

(14 marks)

Q5 (a) Given $f(x) = \sin x$. Approximate $\int_0^{\pi/4} (f'(x))^2 dx$

by using

(i) 2- point Gauss – quadrature.

(ii) 3- point Gauss – quadrature.

(13 marks)

(b) Given

$$A = \begin{pmatrix} 0.04 & 0.01 & -0.01 \\ 0.2 & 0.5 & -0.2 \\ 1 & 2 & 4 \end{pmatrix}.$$

By taking $v^{(0)} = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}^T$, calculate the largest eigenvalue and its corresponding eigenvector by using power method.

Calculate until $|m_{k+1} - m_k| < 0.005$

(7 marks)

Q6 (a) The initial-value problem

$$y' = 4e^{0.8x} - 0.5y\text{, with } y(0) = 2,$$

has unique solution $y(4) = 75.339$. Approximate the solution at $x = 4$ using the fourth-order Runge-Kutta method with the same step size $h = 1$ and compute the percentage relative error.

(8 marks)

(b) Solve the boundary-value problem,

$$y'' + xy = x^3 - \frac{4}{x}, \quad 1 \le x \le 2,$$

with boundary conditions, $4y(1) + y'(1) = 0$, and $3y(2) + 2y'(2) = 0$. Derive the system of linear equations in matrix-vector form by finite difference method (**do not solve the system**). Use $h = \Delta x = 0.2$.

(12 marks)

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Formulae

Nonlinear Equations

Secant method: $$x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)},$$

$i = 0,1,2,.............$

System of linear equation

Gauss-Seidel iteration method: $$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}$$

$i = 1,2,3,.......,n$

Interpolation

Natural Cubic Spline:

$h_k = x_{k+1} - x_k$

$d_k = \dfrac{f_{k+1} - f_k}{h_k}$ $\quad k = 0,1,2,3,4,.........,n-2$

$b_k = 6(d_{k+1} - d_k),$

Consider boundary condition of spline.

$m_0 = 0, m_n = 0$

$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k,$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x - x_k)$$

Numerical Differentiation and Integration

3-point forward difference formula:

$$f'(x) = \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$$

3-point central difference formula:

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

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5-point difference formula:

$$f''(x) = \frac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$$

Gauss-Quadrature:

For $\int_a^b f(x)dx$, $\quad x = \frac{(b-a)t+(b+a)}{2}$

2-point Gauss-Quadrature

$$\int_a^b f(x)dx = g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

3-point Gauss-Quadrature

$$\int_a^b f(x)dx = \frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)$$

Eigenvalue

Power Method: $\quad v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,......$

Ordinary-Differential Equation

Initial-Value Problem:

Fourth-order Runge-Kutta Method

$$y_{i+1} = y_i + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] \quad \text{where}$$

$$k_1 = h\, f(x_i, y_i),$$

$$k_2 = h\, f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right),$$

$$k_3 = h\, f\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right),$$

$$k_4 = h\, f(x_i + h, y_i + k_3),$$

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Boundary value problem

Finite difference method:

$$y_i' = \frac{y_{i+1} - y_{i-1}}{2h}, \quad y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Partial differential equation

Heat Equation: Finite difference method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

Wave equation: Finite difference method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Finite element method

$KT = F_b - F_l,$

where

$$K = A = \int_p^q \frac{dN_i}{dx}\frac{dN_i}{dx}\,dx, \quad T = T_i, \quad F_b = \left[N_i \frac{dT}{dx}\right]_p^q, \quad F_l = -\int_p^q N_i\, Q(x)\,dx$$