UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2009/2010

SUBJECT	:	ENGINEERING MATHEMATICS II
CODE	:	BSM 1923
COURSE	:	2 BFF / BDP
DATE	:	NOVEMBER 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B.**

THIS EXAMINATION PAPER CONSISTS OF 8 PAGES

PART A

Q1 A periodic function is defined as

$$f(x)=\begin{cases}\frac{1}{2}\pi + x, & -\pi < x \le 0,\\ \frac{1}{2}\pi - x, & 0 < x < \pi,\end{cases}$$
$$= f(x+2\pi).$$

(a) Sketch a graph of $f(x)$ from -3π to 3π to determine whether the function is even, odd or neither.

(4 marks)

(b) Show that the Fourier series corresponding to the function is

$$f(x)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2}.$$

(12 marks)

(c) Hence, show that by substituting an appropriate value of x

$$\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{3^2}+\frac{1}{3^2}+\cdots.$$

(4 marks)

Q2 A uniform rod of length 20 cm is fully insulated along its sides. The initial temperature at any point P, a distance of x units from the first end, on the rod is $f(x)$. The temperature at both ends is held fixed at a constant temperature $0°C$. The temperature $u(x,t)$ at time t is

$$\frac{\partial u}{\partial t}=4\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 20, \; t > 0, \dots\dots\dots\dots\dots\dots\dots\dots(1)$$

where k is a constant.

(a) Verify that for arbitrary constants A, B and $\lambda > 0$,

$$u(x,t)=\left(A\cos\lambda x + B\sin\lambda x\right)e^{-4\lambda^2 t}, \dots\dots\dots\dots\dots\dots\dots\dots(2)$$

satisfies (1).

(4 marks)

(b) Fill in the blanks for Q, R and S.

Boundary Conditions: $\left.\begin{aligned} u(0,t) &= \underline{\quad Q \quad}, \\ u(20,t) &= \underline{\quad R \quad}, \end{aligned}\right\}$ $(t>0)$.

Initial Condition: $u(x,0) = \underline{\quad S \quad}$, $(0<x<20)$.

(3 marks)

(c) By substituting one of the boundary conditions into (2), show that

$$u(x,t) = Be^{-4\lambda^2 t}\sin\lambda x.$$

(4 marks)

(d) By applying the second boundary condition, show that

$$\lambda = \frac{n\pi}{20}, \quad n = 1, 2, 3, \ldots,$$

and therefore, by superposition principle, the general solution is

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-\frac{n^2\pi^2 t}{100}} \sin\left(\frac{n\pi x}{20}\right). \quad \ldots\ldots\ldots\ldots (3)$$

(5 marks)

(e) Compute B_n in (3) if the initial temperature $f(x) = 5\sin\frac{\pi x}{4}$.

(4 marks)

PART B

Q3 (a) Show that

$$\frac{y'}{y}=\frac{1}{3x}-\frac{x}{3y^2}$$

is a homogeneous differential equation. Then by using substitution $y=vx$, find the particular solution if $y(1)=1$.

(7 marks)

(b) Obtain the general solution for the differential equation

$$(\cos x + x\cos y - y)\,dy + (\sin y - y\sin x)\,dx = 0.$$

(7 marks)

(c) In a hostel of 1000 population, two students were found to have contracted H1N1 flu after returning home from a mid-semester break. If the flu propagates throughout the population according to the differential equation

$$\frac{dy}{dt}=0.005\left(1000-y\right),$$

where $y(t)$ denotes the number of students who have contracted the disease by time t (in days), how long does it take before 25% of the students have had the disease?

(6 marks)

Q4 (a) Solve the differential equation

$$y''-2y'+y=\frac{e^x}{1+x^2}.$$

$\left[\text{Hint : } \int\frac{1}{1+x^2}\,dx=\tan^{-1}\left(x\right)+c\right].$

(10 marks)

(b) Given a non-homogeneous second order ordinary differential equation as

$$y''+9y'=e^{-9x}+\cos\left(2x\right).$$

Find the general solution for the differential equation by using the method of undetermined coefficient.

(10 marks)

Q5 (a) Show that

(i) $\mathcal{L}\{\cos t-(\cos t)H(t-\pi)\}=\frac{s}{s^2+1}\left(1+e^{-\pi s}\right)$,

(ii) by using convolution theorem

$$\mathcal{L}^{-1}\left\{\frac{s}{\left(s^2+1\right)^2}\right\}=\tfrac{1}{2}t\sin t.$$

(12 marks)

(b) Hence, use the result in Q5 (a) to solve the initial value problem

$$y''+y=\cos t-(\cos t)H(t-\pi),\quad y(0)=2,\ \ y'(0)=1.$$

(8 marks)

Q6 (a) Show that the half-range cosine series for the function

$$f(x)=\left(x-1\right)^2,\quad (0<x<1).$$

is

$$f(x)=\frac{1}{3}+\frac{1}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos n\pi x}{n^2}.$$

(10 marks)

(b) Hence, show that

(i) $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots$.

(ii) $\frac{\pi^2}{12}=1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots$.

(10 marks)

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$, real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$
$P_n(x)\, e^{\alpha x}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x) \cdot \begin{cases} \cos\beta x \\ \text{or} \\ \sin\beta x \end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)\cos\beta x$ + $x^r(C_n x^n + \cdots + C_1 x + C_0{}'')\sin\beta x$
$Ce^{\alpha x} \cdot \begin{cases} \cos\beta x \\ \text{or} \\ \sin\beta x \end{cases}$	$x^r e^{\alpha x}(p\cos\beta x + q\sin\beta x)$
$P_n(x)e^{\alpha x} \cdot \begin{cases} \cos\beta x \\ \text{or} \\ \sin\beta x \end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x$ + $x^r(C_n x^n + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : ***r*** is the least non-negative integer (***r*** = 0, 1, or 2) which determine such that there is no terms in particular integral $y_n(x)$ corresponds to the complementary function $y_c(x)$.

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Laplace Transforms

$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$	
$f(t)$	$F(s)$
a	$\frac{a}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$\dot{y}(t)$	$sY(s) - y(0)$
$\ddot{y}(t)$	$s^2Y(s) - sy(0) - \dot{y}(0)$

Convolution Theorem

Let $\mathcal{L}\{F(s)\} = f(t)$ and $\mathcal{L}^{-1}\{G(s)\} = g(t)$, then $\mathcal{L}^{-1}\{F(s)G(s)\} = \int_0^t f(u)g(t-u)\,du = f(t) * g(t)$

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Variation of Parameters Method

The general solution for $ay'' + by' + cy = f(x)$ is $y(x) = uy_1 + vy_2$, where

$$u = -\int \frac{y_2 f(x)}{aW} dx + A, \quad v = \int \frac{y_1 f(x)}{aW} dx + B, \quad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Fourier Series

Fourier series expansion of periodic function with period $2L$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\, dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\, dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\, dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

Half range series

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\, dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\, dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\, dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$