UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2009/2010

SUBJECT	:	ENGINEERING MATHEMATICS IV
CODE	:	BSM 3913
COURSE	:	2 BDD/BEE/BFF 3 BDD/BEE/BFF
DATE	:	APRIL 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **TWO (2)** QUESTIONS IN **PART B** ALL CALCULATIONS AND ANSWERS MUST BE IN THREE (3) DECIMAL PLACES EXCEPT Q3.

THIS EXAMINATION PAPER CONSISTS OF 8 PAGES

PART A

Q1 (a) The air pressure $p(x,t)$ in an organ pipe is governed by the wave equation

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}, \quad 0 < x < l, \quad 0 < t$$

where l is the length of the pipe and c is a physical constant. When the pipe is open, the boundary conditions are

$$p(0,t) = p_0 \text{ and } p(l,t) = p_0, \quad 0 < t.$$

Assume that the length, l and physical constant, c are 1 respectively and the initial conditions are

$$p(x,0) = p_0 \cos 2\pi x \text{ and } \frac{\partial p}{\partial t}(x,0) = 0, \quad 0 \le x \le 1.$$

Use Finite-Difference method to approximate the pressure for an open pipe with $p_0 = 0.9$ for $t = 0.1$, 0.2 and 0.3. Use $\Delta x = 0.25$ and $\Delta t = 0.1$

(10 marks)

(b) Use the finite-difference method to approximate the solution to the elliptic partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4, \quad 0 < x < 1, \quad 0 < y < 2,$$
$$u(x,0) = x^2, \quad u(x,2) = (x-2)^2, \quad 0 \le x \le 1,$$
$$u(0,y) = y^2, \quad u(1,y) = (y-1)^2, \quad 0 \le y \le 2.$$

Use $\Delta x = \Delta y = 0.5$ and find an absolute error from the solution you have obtained if the actual solution is $u(x,y) = (x-y)^2$.

(15 marks)

Q2 Consider a fin of length 5 unit has the constant width along the length and equal to 1.25 unit. The heat flow equation is:

$$\frac{d}{dx}\left(Ak\frac{dT}{dx}\right)+Q(x)=0$$

with A is the cross-sectional area, k is the thermal conductivity, T is the temperature at length x and Q is the heat supply per unit time and per unit length.

Find the system of linear equations to determine the temperature at each nodal point, $x = 1.25$, 2.50, 3.75, 5.00 and if the cross-sectional area 30 unit, thermal conductivity of the material of the fin is 10 and Q is 10 unit. Let the temperature when $x = 0$ is 0 unit and the heat flux, $-k\left.\frac{dT}{dx}\right|_{x=5} = 10$ unit. PLEASE DO NOT SOLVE THE SYSTEM.

(25 marks)

PART B

Q3 FOR Q3 (a) and Q3 (b), CALCULATIONS AND ANSWERS MUST BE IN 6 DECIMAL PLACES.

(a) Experimental data shows that the viscosity of sulfuric acid is related to its concentration. It was obtained by using a viscometer which measures the variations in viscosity with concentration. The following data in **Table Q3(a)** below represents the viscosity of sulfuric acid, in millipascal-seconds (centipoises), as a function of concentration, in mass percent.

Table Q3(a)

Concentration, x (mass percent)	0	20	40	60	80	100
Viscosity, $f(x)$ (mPa/sec.)	0.89	1.4	2.51	a	17.4	24.2

By using Natural Cubic spline polynomial, estimate:

(i) the value of viscosity for the sulfuric acid a.

(ii) the viscosity when the concentration for the sulfuric acid is 10%.

(18 marks)

(b) Based on the cubic spline function, S that you obtained in part (a), find:

(i) the rate of change for viscosity of sulfuric acid at $x = 3$ by using the 2-points backward, 3-points central and 5-points difference formula.

(ii) $\left.\frac{d^2S}{dx^2}\right|_{x=17}$ by using 3-point central difference formula. Hence, interpret your result.

For both (i) and (ii), use $h = 0.1$.

(7 marks)

Q4 (a) A simple pendulum consists of a mass that swings in a vertical plane at the end of a massless rod of length L, as shown in the accompanying **Figure Q4(a)**. Suppose that a simple pendulum is displaced through an angle θ_0 and released from rest. It can be shown that in the absence of friction, the time T required for the pendulum to make one complete back-and-forth swing, called the period, is given by

$$T = 4\sqrt{\frac{L}{g}}\int_0^{\pi/2}\frac{1}{\sqrt{1-k^2\sin^2\phi}}d\phi$$

where $k = \sin(\theta_0/2)$.

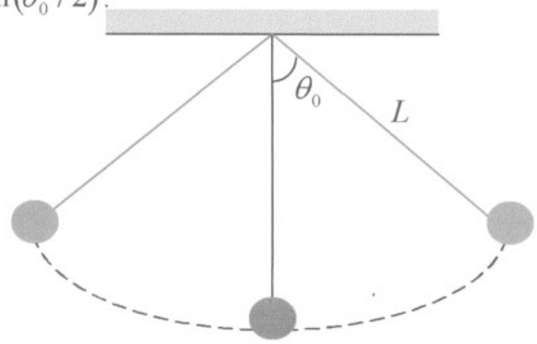

Figure Q4(a)

(i) Estimate the period of a simple pendulum for which $L = 1.5\,\text{ft}$, $\theta_0 = \frac{\pi}{9}$, and $g = 32\ \text{ft/s}^2$ by using 2-point Gauss quadrature.

(ii) Find the relative error if the exact value of the period of a simple pendulum above is 1.37 s.

(12 marks)

(b) Solve $e^{x+y}\frac{dy}{dx} = x$ at $x = 0(0.2)1$ by Euler's method with initial condition $y(0) = 0$.

(5 marks)

(c) Given the boundary value problem,

$$-u'' + \pi^2 u = 2\pi^2\sin(\pi x), \text{ in the interval } [0,\ 1],$$

with the boundary conditions

$$u(0) = u(1) = 0.$$

Derive the system of linear equations, in matrix-vector form using the finite difference method by taking $\Delta x = h = 0.2$. PLEASE DO NOT SOLVE THE SYSTEM.

(8 marks)

Q5 FOR QUESTIONS Q5 (a) AND Q5 (b) BELOW, DO YOUR CALCULATIONS UP TO 3 ITERATIONS ONLY.

(a) An engineer needs $4800m^3$, $5810m^3$ and $5690m^3$ of sand, fine gravel and coarse gravel, respectively, at a construction site. There are three sources where these materials can be obtained and the composition of the material from these sources is shown on **Table Q5(a)**.

Table Q5(a)

	%Sand	%Find gravel	%Coarse Gravel
Source 1	52	30	18
Source 2	20	50	30
Source 3	25	20	55

How many cubic meters must be hauled from each source in order to meet the engineer's needs? Use Gauss Seidel Iteration method to solve this problem.

(10 marks)

(b) Find the smallest (in absolute value) eigenvalue and its corresponding eigenvector for matrix A below by shifted power method. Let $v = [1 \quad 1 \quad 1]^T$ as initial guess for the eigenvector.

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

(15 marks)

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FORMULAS

System of linear equations

Gauss-Seidel iteration method:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, i = 1, 2, \ldots, n$$

Interpolation

Natural Cubic Spline:

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\} \qquad k = 0,1,2,3,\ldots,n-1$$

$$b_k = 6(d_{k+1} - d_k), \qquad k = 0,1,2,3,\ldots n-2$$

Consider boundary condition of spline.

$$m_0 = 0, m_n = 0$$

$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, k = 0,1,2,\ldots n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6} h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6} h_k\right)(x - x_k)$$

where $k = 0,1,2,3,\ldots,n-1$

Eigen value

Power Method: $\mathbf{v}^{(k+1)} = \frac{1}{m_{k+1}} A \mathbf{v}^{(k)}, \qquad k = 0,1,2,\ldots..$

Shifted Power Method: $A_{shifted} = A - \lambda_l I$

Numerical differentiation and integration

First derivatives:

2-point forward difference: $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x) - f(x-h)}{h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

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3-point central difference: $f'(x) \approx \dfrac{f(x+h)-f(x-h)}{2h}$

5-point difference: $f'(x) \approx \dfrac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

Second derivatives:

3-point central difference: $f''(x) \approx \dfrac{f(x+h)-2f(x)+f(x-h)}{h^2}$

5-point difference: $f''(x) \approx \dfrac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$

Integration:

Gauss quadrature:

For $\int_a^b f(x)dx$, $\quad x = \dfrac{(b-a)t+(b+a)}{2}$

2-points: $\int_{-1}^{1} f(x)dx \approx g\left(-\dfrac{1}{\sqrt{3}}\right)+g\left(\dfrac{1}{\sqrt{3}}\right)$

3-points: $\int_{-1}^{1} f(x)dx \approx \dfrac{5}{9}g\left(-\sqrt{\dfrac{3}{5}}\right)+\dfrac{8}{9}g(0)+\dfrac{5}{9}g\left(\sqrt{\dfrac{3}{5}}\right)$

Ordinary differential equations (ODE)

Initial value problems:

Euler's method: $y(x_{i+1}) = y(x_i) + hy'(x_i)$

Second order Taylor series method: $y(x_{i+1}) = y(x_i) + hy'(x_i) + \dfrac{h^2}{2!}y''(x_i)$

Improved Euler's method (Mid point method):

$y_{i+1} = y_i + k_2$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \dfrac{h}{2}, y_i + \dfrac{k_1}{2})$

Boundary value problems:

Finite difference method:

$y_i' \approx \dfrac{y_{i+1}-y_{i-1}}{2h}$ $\quad y_i'' \approx \dfrac{y_{i+1}-2y_i+y_{i-1}}{h^2}$

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Partial differential equations

Heat equation- Finite difference method:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

Wave equation- Finite difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = c^2 \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

Laplace's equation-Finite difference method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = 0 \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Poisson equation-Finite difference method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = f_{i,j}$$

Finite element method

$KT = F_b - F_l$

where $K_{ij} = \int_p^q A(x)k(x)\frac{dN_i}{dx}\frac{dN_j}{dx}dx$ is stiffness matrix,

$$T = T_i$$

$$F_b = \left[N_i A(x)k(x)\frac{dT}{dx}\right]_p^q$$

$$F_l = -\int_p^q N_i Q(x)dx$$