UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2009/2010

SUBJECT	:	ENGINEERING MATHEMATICS III
CODE	:	BSM 2913
COURSE	:	1 BDD / BDI / BEE / BEI / BFF /BFI 2 BDD / BDI / BEE / BEI / BFF /BFI 3 BDD / BDI / BEE / BEI / BFF /BFI
DATE	:	APRIL 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 6 PAGES

PART A

Q1 (a) Show that the vector field $\mathbf{F}(x, y, z) = 2x(y^2 + z^3)\mathbf{i} + 2yx^2\mathbf{j} + 3x^2z^2\mathbf{k}$ is conservative. Find its scalar potential function $\phi(x, y, z)$ in moving a particle from $(-1, 2, 1)$ and $(2, 3, 4)$.

(10 marks)

(b) Verify the Green's theorem for $f(x, y) = e^{-x}\sin y$, $g(x, y) = e^{-x}\cos y$ and C is the square with vertices at $(0,0), (\pi/2, 0), (\pi/2, \pi/2), (0, \pi/2)$.

(10 marks)

Q2 (a) Evaluate

$$\iint_\sigma \nabla \times \mathbf{F} \cdot \mathbf{n}\, dS$$

with $\mathbf{F}(x, y, z) = (2x - y)\mathbf{i} - (yz^2)\mathbf{j} + (y^2z)\mathbf{k}$, and σ is the surface of a paraboloid $z = 4 - x^2 - y^2$ located above the xy-plane. Check your result by using Stokes's theorem.

(10 marks)

(b) Let $\mathbf{F}(x, y, z) = x\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$ and σ is the boundary of the solid inside a cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 2$. Use Gauss's theorem to

$$\iint_\sigma \mathbf{F} \cdot \mathbf{n}\, dS$$

with $\mathbf{n}$ as the unit normal vector oriented pointing outwards.

(10 marks)

PART B

Q3 (a) If $z = \dfrac{x+y}{\sqrt{x^2+y^2}}$, find $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$.

(6 marks)

(b) A box with height h has a square base with length x. The error in measuring the side of the base is 1%, whereas for the height is 2%. Approximate the maximum percentage error in calculating the volume.

(6 marks)

(c) Obtain the local extremums and saddle point (if exists) for the function

$$f(x,y) = 2x^4 + y^2 - 12xy.$$

(8 marks)

Q4 (a) Use polar coordinates to find volume of the part of the sphere of radius 3 that is left after drilling the cylindrical hole of radius 2 through the center.

(7 marks)

(b) Given a lamina with density $\rho(x,y,z) = 4$ enclosed by $z = x^2 + y^2$ and $z = 4$. Find

(i) its mass.

(ii) moment of mass about xy-axis.

(iii) the coordinate $\overline{z}$ of the center of gravity.

(13 marks)

Q5 (a) Sketch a vector valued function $\mathbf{r}(t)$ that represents the curve of intersection of the cylinder $x^2 + z^2 = 4$ and the plane $y = 3$. Thus, find $\mathbf{r}(t)$.

(4 marks)

(b) An object moves with an acceleration $\mathbf{a}(t) = (t+1)^{-2}\,\mathbf{j} - e^{-2t}\mathbf{k}$. Find its velocity $\mathbf{v}(t)$ and its speed at $t = 0.5$ if given $\mathbf{v}(0) = 3\mathbf{i} - \mathbf{j}$.

(10 marks)

(c) Find the arc length of a position of the circular helix $\mathbf{r}(t) = a\cos t\mathbf{i} + a\sin t\mathbf{j} + \sqrt{1-a^2}\,t\mathbf{k}$ from $t = 0$ to $t = 2\pi$.

(6 marks)

Q6 (a) Find the directional derivative of $f(x,y)=x^2y^3+xy$ at $(2,1)$, in the direction of a unit vector $\hat{\mathbf{b}}=\cos\theta\,\mathbf{i}+\sin\theta\,\mathbf{j}$ which makes angle of $\dfrac{\pi}{3}$ with x-axis.

(6 marks)

(b) Evaluate

$$\int_C (x+y)dx - x^2dy + (y+z)dz,$$

where C is $x^2=4y,\ z=x,\ 0\le x\le 2.$

(4 marks)

(c) Evaluate the surface integral

$$\iint_S y\,dA,$$

where S is the portion of the cylinder $x=6-y^2$ in the first octant bounded by the planes $x=0,\ y=0,\ \ z=0$ and $z=8$.

(10 marks)

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Formulae

Polar coordinates: $x = r\cos\theta,\ y = r\sin\theta$ and $x^2 + y^2 = r^2$

$$\iint_R f(x,y)\,dA = \iint_R f(r,\theta)\, r\,dr\,d\theta$$

Cylindrical coordinates: $x = r\cos\theta,\ y = r\sin\theta, z = z$ and $x^2 + y^2 = r^2$

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\, r\,dz\,dr\,d\theta$$

Spherical coordinates: $x = \rho\cos\theta\sin\phi,\ y = \rho\sin\theta\sin\phi,\ z = \rho\cos\phi,\ \rho^2 = x^2 + y^2 + z^2,$
$0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(\rho,\phi,\theta)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

The directional derivatives, $D_u f(x,y) = \left(f_x\,\mathbf{i} + f_y\,\mathbf{j}\right)\cdot\mathbf{u}$; The **gradient** of $\phi = \nabla\phi$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

The **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\dfrac{\partial P}{\partial y} - \dfrac{\partial N}{\partial z}\right)\mathbf{i} - \left(\dfrac{\partial P}{\partial x} - \dfrac{\partial M}{\partial z}\right)\mathbf{j} + \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$.

The **unit tangent vector**, $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **principal unit normal vector**, $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

Curvature, $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

Green's Theorem:

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

Gauss's Theorem:

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_G \nabla\cdot\mathbf{F}\,dV$$

Stokes's Theorem:

$$\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS = \int_C \mathbf{F}\cdot dr$$

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Arc Length of Plane Curve and Space Curve

For a plane curve, $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$ on an interval $[a, b]$, the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\| \, dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} \, dt.$$

For a space curve, $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathrm{k}$ on an interval $[a, b]$, the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\| \, dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} \, dt.$$