UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2009/2010

SUBJECT	:	ENGINEERING MATHEMATICS II
CODE	:	BSM 1933
COURSE	:	1BEE/2BEE/3BEE/4BEE
DATE	:	APRIL / MAY 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 8 PAGES

PART A

Q1 Given

$$y'' + y = 0.$$

(a) By assuming $y = \sum_0^\infty c_m x^m$, show that the differential equation

$$y'' + y = 0$$

can be expressed as

$$\sum_{n=0}^{\infty} [(n+2)(n+1)c_{n+2} + c_n] x^n = 0.$$

(6 marks)

(b) Hence, by shifting the indices, show that the recurrence relation is given by

$$c_{n+2} = -\frac{c_n}{(n+1)(n+2)}, \qquad n = 0, 1, 2, 3, ...$$

(2 marks)

(c) Then, deduce the coefficient of series, c_n, *for* $n = 0, 1, 2, 3, ...$ in terms of c_0 and c_1.

(6 marks)

(d) Hence, show that the general solution of the differential equation is

$$y(x) = c_0 \cos x + c_1 \sin x.$$

(6 marks)

Q2 (a) A periodic function is defined as

$$f(x)=\begin{cases}\frac{1}{2}\pi+x, & -\pi<x\le 0\\ \frac{1}{2}\pi-x, & 0<x\le\pi,\end{cases}$$

$$=f(x+2\pi).$$

By substituting appropriate initial condition, show

$$1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots\cdots=\frac{\pi^2}{8}.$$

(12 marks)

(b) Find the Fourier transform of $f(x)$, defined as

$$f(x)=\begin{cases}1, & |x|<a\\ \\ 0, & |x|>a\end{cases}$$

Hence, find the value of $\int_0^\infty \frac{\sin x}{x}dx$ (8 marks)

PART B

Q3 (a) By eliminating the constant A from the equation

$$y = 6x^2 - \frac{A}{2x}$$

form a linear first order ordinary differential equation.

(6 marks)

(b) Consider the following differential equation

$$(x^5 y^{k+1} + 2y\cos 2x)dx + (2x^6 y^k + \sin 2x + 3)dy = 0.$$

(i) Determine the value of k such that the differential equation above is exact.

(ii) Obtain the general solution of the exact equation in (b)(i).

(14 marks)

Q4 (a) Find the solution for the differential equation

$$y'' - 14y' + 49 = \frac{3e^{7x}\ln x}{2}$$

(11 marks)

(b) Given an RLC-circuit in with $R = 25\Omega$, $L = 5\,\text{mH}$, $C = 2\,\text{F}$, and emf source is constant.

(i) By using Kirchhoff's Current Law (KCL), show the circuit can be modeled by

$$V''(t) + 0.02V'(t) + 100V(t) = 0.$$

(ii) Find the general solution to the second-order differential equation in (b)(i).

(iii) Find the particular solution of $V(t)$ given that $V = 6$ and $V'(t) = 0$, when $t = 0$.

(9 marks)

Q5 Given the network circuit in **Figure Q5**.

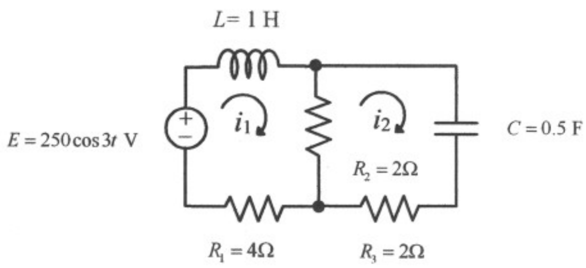

Figure Q5

(a) Show that the network circuit can be modeled by the following system of first-order differential equation

$$\begin{pmatrix} i_1' \\ i_2' \end{pmatrix} = \begin{pmatrix} -6 & 2 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} 250\cos(3t) \\ 125\cos(3t) \end{pmatrix}.$$

(6 marks)

(b) Find the general solution to the above system of first-order differential equation by using method of undetermined coefficients.

(14 marks)

Q6 (a) Find the Laplace transform for the following functions:

(i) $f(t) = \sin 2t \sin 3t$ (ii) $f(t) = t\sin^2 t$ (8marks)

(b) Find the Inverse Laplace transform of $\dfrac{s+2}{s^2-4s+13}$ (4marks)

(c) Given the *R-L* circuit with source of emf, $E(t)$ as in **Figure Q6(c)** below.

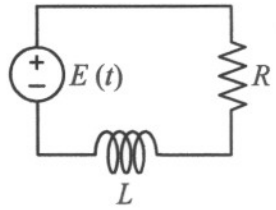

Figure Q6(c)

The circuit has inductance $L = 2H$, resistance $R = 10\,\Omega$, electromotive force $E(t) = 2e^t V$ which is initially at rest. Let $i(t)$ is the current flowing in the circuit.

(i) Show that the mathematical model for the *RL* circuit is given by

$$\frac{di(t)}{dt} + 5i(t) = e^t .$$

(ii) Using Laplace Transform to find the current, $i(t)$, flowing in the circuit at time t.

(8 marks)

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Formulas

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients for system of first order linear differential equations

For non-homogeneous for system of first order linear differential equations $\mathbf{Y}'(x) = A\,\mathbf{Y}(x) + \mathbf{G}(x)$, the particular solution $\mathbf{Y}_p(x)$ is given by:

$\mathbf{G}(x)$	$\mathbf{Y}_p(x)$	$\mathbf{G}(x)$	$\mathbf{Y}_p(x)$
$\mathbf{u}$	$\mathbf{a}$	$\mathbf{u}e^{\lambda x}$	$\mathbf{a}e^{\lambda x}$
$\mathbf{u}x + \mathbf{v}$	$\mathbf{a}x + \mathbf{b}$	$\mathbf{u}\cos\alpha x$ or $\mathbf{u}\sin\alpha x$	$\mathbf{a}\sin\alpha x + \mathbf{b}\cos\alpha x$

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
e^{at}	$\frac{1}{s-a}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\sinh at$	$\frac{a}{s^2-a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$y(t)$	$Y(s)$
$e^{at}f(t)$	$F(s-a)$	$y'(t)$	$sY(s) - y(0)$

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$t^n f(t)$, $n=1,2,3,...$	$(-1)^n \frac{d^n}{ds^n}F(s)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

Electrical Formula

1. Voltage drop across resistor, R (Ohm's Law): $v_R = iR$
2. Voltage drop across inductor , L (Faraday's Law): $v_L = L\frac{di}{dt}$
3. Voltage drop across capacitor , C (Coulomb's Law): $v_C = \frac{q}{C}$ or $i = C\frac{dv_C}{dt}$
4. The relation between current, i and charge, q: $i = \frac{dq}{dt}$.

Fourier Series

Fourier series expansion of periodic function with period $2L/2\pi$	Half Range series
$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$	$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$
$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$	$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$	$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$
$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$	$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$

Table of Fourier Transform $\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt$

$f(t)$	$F(\omega)$	$f(t)$	$F(\omega)$
$\delta(t)$	1	$\mathrm{sgn}(t)$	$\frac{2}{i\omega}$
$\delta(t-\omega_0)$	$e^{-i\omega_0\omega}$	$H(t)$	$\pi\delta(\omega)+\frac{1}{i\omega}$
1	$2\pi\delta(\omega)$	$e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{1}{\omega_0+i\omega}$
$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$t^n e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{n!}{(\omega_0+i\omega)^{n+1}}$
$\sin(\omega_0 t)$	$i\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	$e^{-at}\sin(\omega_0 t)H(t)$ for $a>0$	$\frac{\omega_0}{(a+i\omega)^2+\omega_0^{\ 2}}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	$e^{-at}\cos(\omega_0 t)H(t)$ for $a>0$	$\frac{a+i\omega}{(a+i\omega)^2+\omega_0^{\ 2}}$
$\sin(\omega_0 t)H(t)$	$\frac{\pi}{2}i[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$		
$\cos(\omega_0 t)H(t)$	$\frac{\pi}{2}[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]+\frac{i\omega}{\omega_0^2-\omega^2}$		