UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2009/2010

SUBJECT	:	ENGINEERING MATHEMATICS II
CODE	:	BSM 1923
COURSE	:	1 BFF / 3 BFF / 4 BFF / 1 BDD / 2 BDD / 3 BDD / 4 BDD
DATE	:	APRIL 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 6 PAGES

PART A

Q1 A periodic function $f(x)$ is defined by

$$f(x)=\begin{cases}-\sin\left(\dfrac{\pi x}{2}\right), & -1<x<0\\ \sin\left(\dfrac{\pi x}{2}\right), & 0<x<1\end{cases}$$
$$= f(x+2).$$

(a) Sketch the graph of the function over the interval $-3<x<3$.

(3 marks)

(b) Show that $2\int_0^1 \sin\left(\frac{\pi x}{2}\right)\cos n\pi x\,dx=\frac{1}{\pi}\left[\frac{4}{1-4n^2}\right]$.
$\left[\text{Hint}: 2\sin A\cos B=\sin(A+B)+\sin(A-B).\right]$

(7 marks)

(c) Show that the Fourier series expansion of $f(x)$ is

$$f(x)=\frac{2}{\pi}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos n\pi x}{4n^2-1}.$$

(7 marks)

(d) Using the value $x=0$ in part (b), find the sum of the following series

$$\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\cdots+\frac{1}{4n^2-1}+\cdots$$

(3 marks)

Q2 (a) Show that the function $u(x,t)=Ae^{2x}\sin(2x+t)-Be^{-2x}\sin(2x-t)$ is a solution of $\dfrac{\partial^2 u}{\partial x^2}=8\dfrac{\partial u}{\partial t}$, where A and B are constants.

(6 marks)

(b) A stretched spring of length l = 10 cm with both ends fixed is set oscillating. The initial state is given by a function, $f(x)=1$. The string has been released with initial velocity $g(x)=x$. By applying the method of separation of variables to the wave equation, $u_{xx}=u_{tt}$, the subsequent displacement of the spring is given by

$$u(x,t)=\sum_{n=1}^{\infty}\left[A_n\cos\left(\frac{n\pi}{10}\right)t+B_n\sin\left(\frac{n\pi}{10}\right)t\right]\sin\left(\frac{n\pi}{10}\right)x,$$

where A_n and B_n are the Fourier coefficients.

(i) List the initial and boundary conditions.

(ii) Find the values of A_n and B_n.

$$\left[\text{Hint}: A_n=\frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi}{l}\right)x\,dx,\ B_n=\frac{2}{n\pi c}\int_0^l g(x)\sin\left(\frac{n\pi}{l}\right)x\,dx.\right]$$

(14 marks)

PART B

Q3 (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{y^3}{x(x^2 + y^2)}, \quad y(1) = 1.$$

(10 marks)

(b) The equation of motion is governed by

$$x'' + \frac{a}{M}x' + \frac{k}{M}x = \frac{F(t)}{M}$$

where M is a mass and k is a spring constant. Suppose that a mass of 2 kg is suspended from a spring with a known spring constant of 4 N/m and allowed to come to rest. It is then set in motion by giving it an initial velocity of 2 m/s. Find the position of the mass at any time if the magnitude of the external force, $F(t) = 2t$ and the air resistance, $a = 6$.

(10 marks)

Q4 (a) Solve

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + y = \frac{1}{xe^x}.$$

(11 marks)

(b) The equation of a free fall model is governed by

$$m\frac{dv}{dt} = mg - F_A$$

where m is a mass, g is the acceleration due to gravity and F_A is an air resistance. Suppose that a 10kg ballast bag is dropped from a hot air balloon which is at an altitude of 342 meter above the ground. If the initial velocity of ballast bag is 5 meter per second and air resistance is given by $2v$, determine the velocity of the ballast bag. [Take $g = 9.8\,\text{m/s}^2$]

(9 marks)

Q5 (a) Find the Laplace transforms of

$$f(t) = e^{3t}(e^2 + e^{-5t}) - \cos(\pi - t) + \sinh t\,\delta(t - \pi).$$

(5 marks)

(b) Show that

$$\mathcal{L}^{-1}\left\{\frac{3}{s^3 - 3s^2}\right\} = \frac{1}{3}(e^{3t} - 1 - 3t).$$

(6 marks)

(c) Consider the initial value problem

$$y'' - 3y' = 6H(t-4)$$

subject to the conditions $y(0) = 0$ and $y'(0) = 0$.

(i) By Laplace transforms, show that

$$\mathcal{L}\{y(t)\} = 2e^{-4s}\left(\frac{3}{s^3 - 3s^2}\right).$$

(ii) Then, determine the solution of the initial value problem by using the result in **Q5 (b)**.

(9 marks)

Q6 (a) Find the Laplace transforms of the following functions:

(i) $f(t) = e^{3t}\sin 3t \sin t + t\sin 3t \cos 3t.$

(ii) $f(t) = (t-3)(t+5)H(t-3).$

(12 marks)

(b) Consider the periodic function

$$f(t) = \begin{cases} e^t, & 0 \le t < 1 \\ 3, & 1 \le t < 2 \end{cases}$$

$$f(t) = f(t+2).$$

(i) Sketch the graph of $f(t)$ for $0 \le t \le 6$.

(ii) Find the Laplace transform of $f(t)$.

(8 marks)

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Formulae
Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$
$P_n(x)\, e^{\alpha x}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\cdot\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + \cdots + C_1 x + C_0{}'')\sin\beta x$
$Ce^{\alpha x}\cdot\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(p\cos\beta x + q\sin\beta x)$
$P_n(x)e^{\alpha x}\cdot\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W}dx, \quad u_2 = \int \frac{y_1 f(x)}{W}dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1, 2, 3, \ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \dfrac{1}{1-e^{-sT}}\displaystyle\int_0^T e^{-st} f(t)\,dt, \quad s>0.$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right] \quad \text{where} \quad \begin{aligned} a_0 &= \frac{1}{L}\int_{-L}^{L} f(x)\,dx \\ a_n &= \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx \\ b_n &= \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx \end{aligned}$$