UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2009/2010

SUBJECT	:	ENGINEERING MATHEMATICS I
CODE	:	BSM 1913
COURSE	:	1 BFF/BDD/BEE 2 BFF/BDD 4 BDD/BEE
DATE	:	APRIL 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B.**

THIS EXAMINATION PAPER CONSISTS OF 6 PAGES

PART A

Q1 (a) Evaluate $\int \frac{dx}{x\sqrt{4+x^{16}}}$.

(6 marks)

(b) Find the surface area generated when $y=\sqrt[3]{3x}$ from $y=-1$ to $y=0$ is rotated 360° about the y-axis.

(9 marks)

(c) Find the curvature for $x=\cos t,\ y=\ln 2t$ at $t=\pi$.

(5 marks)

Q2 (a) Show that the Maclaurin series for $f(x)=\sin x$ is

$$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$$

Then,

(i) evaluate $\int_0^1 \sin x^3\,dx$.

(ii) find the first six terms of a series for $\cos x$ and also $2\cos x\sin x$.

(12 marks)

(b) Determine whether the series

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt[3]{n}}$$

is absolutely convergent, conditionally convergent or divergent.

(8 marks)

PART B

Q3 (a) Evaluate the limits below.

(i) $\lim_{x \to 7} \dfrac{\frac{1}{x} - \frac{1}{7}}{x - 7}$.

(ii) $\lim_{x \to +\infty} \sqrt{x^2 + 5} - x$.

(iii) $\lim_{x \to 0} (\sin 2x + 1)^{1/x}$.

(14 marks)

(b) Determine whether or not the following function is continuous or not at $x = -2$.

$$f(x) = \begin{cases} \dfrac{x+2}{x^3 + 2x^2 + x + 2}, & x < -2, \\ \dfrac{1}{x} - \dfrac{7}{5x}, & x \geq -2. \end{cases}$$

(6 marks)

Q4 (a) Find $\dfrac{dy}{dx}$ if $y = x^{-2} \sin^2(x^3)$.

(5 marks)

(b) Determine the slope of the curve,

$$x = \frac{t^3}{1+t^2} \quad \text{and} \quad y = \frac{4t+3}{t}$$

when $t = -1$.

(6 marks)

(c) Given $f(x) = \dfrac{x^2}{x^2 + 2}$.

(i) Find the open interval of the function f on which f is concave upward or downward.

(ii) Hence, determine the points of inflection of the function.

(9 marks)

Q5 (a) Integrate the following expression with respect to x.

(i) $\dfrac{x+4}{(x-2)^2}$.

(ii) $\sec^2 x \tan^5 x$.

(10 marks)

(b) Evaluate $\displaystyle\int \frac{2}{\cos^2 x}\,dx$ by using $t = \tan x$ substitution.

(5 marks)

(c) Find $\displaystyle\int \frac{3}{\sqrt{x^2-9}}\,dx$ by using a hyperbolic substitution.

(5 marks)

Q6 (a) Find the radius and interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (2x+1)^{2n}}{(2n+1)9^n}.$$

(12 marks)

(b) Discuss the convergence of the following series.

(i) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{-2n}{n+1}\right)^{6n}$.

(ii) $1+2+4+8+\cdots+2^{n-1}+\cdots$.

(iii) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(n+1)^4}$.

(8 marks)

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Formulae

Indefinite Integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \cosh x\,dx = \sinh x + C$$

$$\int \sinh x\,dx = \cosh x + C$$

$$\int \text{sech}^2 x\,dx = \tanh x + C$$

$$\int \text{csch}^2 x\,dx = -\coth x + C$$

$$\int \text{sech}\, x \tanh x\,dx = -\text{sech}\, x + C$$

$$\int \text{csch}\, x \coth x\,dx = -\text{csch}\, x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C, \ |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1} x + C, \ |x| < 1$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\,dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\,dx = \csc^{-1} x + C, \ |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\,dx = \text{sech}^{-1} |x| + C, \ 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\,dx = \text{csch}^{-1} |x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2}\,dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

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Formulae

TRIGONOMETRIC SUBSTITUTION

$t = \tan \frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$ $\tan x = \dfrac{2t}{1-t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{2dt}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$ $\tan 2x = \dfrac{2t}{1-t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x + \sin^2 x = 1$$
$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$
$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$2\sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$$
$$2\sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$$
$$2\cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$
$$1 - \tanh^2 x = \text{sech}^2 x$$
$$\coth^2 x - 1 = \text{csch}^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$
$$\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left| \dfrac{d^2 y}{dx^2} \right|}{\left[1 + \left(\dfrac{dy}{dx} \right)^2 \right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = 2\pi \int_{x_1}^{x_2} f(x) \sqrt{1 + \left(\frac{d}{dx}[f(x)]\right)^2}\, dx$$

$$S = 2\pi \int_{y_1}^{y_2} g(y) \sqrt{1 + \left(\frac{d}{dy}[g(y)]\right)^2}\, dy$$