UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2009/2010

SUBJECT	:	MATHEMATICS III
CODE	:	DSM 2933
COURSE	:	2 DET/ DEE
DATE	:	NOVEMBER 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND CHOOSE **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 7 PAGES

PART A

Q1 (a) Find the solution of the differential equation

$$\frac{dy}{dx} - y\tan x = 1 + \sin^2 x$$

which satisfies $y\left(\frac{\pi}{2}\right) = 0$.

(10 marks)

(b) By using the undetermined coefficients method, solve

$$y'' - y' - 12y = e^{4x}.$$

(10 marks)

Q2 (a) Solve the homogeneous equation

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy - x^2}$$

if $y = 0$ when $x = 1$.

(10 marks)

(b) Solve $y'' + 2y' + y = e^{-x}\cos x$ by using the variation of parameter method.

(10 marks)

PART B

Q3 (a) The surface area of a sphere is decreasing at the constant rate of 5π cm^2/s. At what rate is the volume decreasing when its radius is 2cm?

(6 marks)

(b) Determine the relative extrema of the function $f(x) = x + \frac{1}{x}$.

(6 marks)

(c) Find $\lim_{x \to 0} (\cos x)^{\frac{1}{x}}$.

(8 marks)

Q4 Evaluate the following integrals.

(a) $\int x \ln \sqrt{x}\, dx$.

(5 marks)

(b) $\int x^3 \sin x^2\, dx$.

(6 marks)

(c) $\int \frac{1}{\sqrt{-x^2 - 2x + 3}}\, dx$.

(6 marks)

(d) $\int_{-1}^{\infty} e^{-2x}\, dx$.

(3 marks)

Q5 (a) Evaluate $\int_0^1 x(x^2+1)^5\, dx$ by using the Simpson's rule with 8 subintervals.

(6 marks)

(b) Find the arc length of the curve $y = \frac{1}{3}\left(2 + x^2\right)^{\frac{3}{2}}$ over the interval $[0, 3]$.

(6 marks)

(c) Find the area of the surface generated by revolving the curve $y = \sqrt{x}$ from $x = 0$ to $x = 4$ about the x-axis.

(8 marks)

Q6 (a) Solve the initial value problem

$$2y'' - 5y' - 3y = 0$$

given that at $x = 0$, $y = 0$ and $y' = 1$.

(8 marks)

(b) Show that the differential equation $(2xy^2 + 2y)dx + (2x^2y + 2x)dy = 0$ is exact. Hence, find the solution.

(6 marks)

(c) Solve $\dfrac{dy}{dx} = \dfrac{y}{x(x+1)}$.

(6 marks)

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Formulas

Differentiation And Integration Formula

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}\csc x = -\csc x \cot x$	$\int \csc x \cot x\, dx = -\csc x + C$
$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C$
$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C$
$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$

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Numerical Integration

Trapezoidal's Rule :

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(x_i) \right]$$

Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a) + f(b) + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f(a+ih) + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f(a+ih) \right]$$

Volume, Arc Length and Surface Area of Revolution

$$V = \pi \int_a^b \left[f(x) \right]^2 \, dx$$

$$V = \pi \int_a^b \left([f(x)]^2 - [g(x)]^2 \right) dx$$

$$V = \pi \int_c^d \left[w(y) \right]^2 \, dy$$

$$V = \pi \int_c^d \left([w(y)]^2 - [v(y)]^2 \right) dy$$

$$V = 2\pi \int_a^b x\, f(x) \, dx$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy} \right)^2} \, dy$$

$$S = 2\pi \int_a^b f(x) \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx$$

$$S = 2\pi \int_a^b g(y) \sqrt{1 + \left(\frac{dx}{dy} \right)^2} \, dy$$

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Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$;
Characteristic equation : $am^2 + bm + c = 0$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$

$f(x)$	$y_p(x)$
$P_n(x) = A_0 + A_1 x + \cdots + A_n x^n$	$x^r(B_0 + B_1 x + \cdots + B_n x^n)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

Variation of Parameters Method for $ay'' + by' + cy = f(x)$

$$y(x) = uy_1 + vy_2$$

$$u = -\int \frac{y_2 f(x)}{aW}\,dx + A \qquad v = \int \frac{y_1 f(x)}{aW}\,dx + B$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$