UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2009/2010

SUBJECT	:	MATHEMATICS III
CODE	:	DSM 2913
COURSE	:	2 DDM / DFA / DFT
DATE	:	NOVEMBER 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **FOUR (4)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 7 PAGES

PART A

Q1 (a) Find the Laplace transform of the following functions.

(i) $t^4 + \sin\sqrt{2}t$.

(ii) $te^{-t}\cos 2t$.

(8 marks)

(b) Given

$$f(t) = \begin{cases} 0, & 0 < t < 3, \\ t, & 2 < t < 5, \\ e^{2t}, & t > 5. \end{cases}$$

(i) Express the function $f(t)$ in the form of the unit step function.

(ii) Find the Laplace transform of $f(t)$.

(9 marks)

(c) By using the first shift theorem, find the inverse Laplace transform of $\dfrac{2s+1}{s^2-2s+10}$.

(5 marks)

Q2 Consider the following initial value problem

$$y'' - 2y' + 5y = -8e^{-t}, \qquad y(0) = 2, \qquad y'(0) = 12.$$

(a) Show $Y(s) = \dfrac{2s}{(s-1)^2+4} + \dfrac{8}{(s-1)^2+4} - \dfrac{8}{(s+1)[(s-1)^2+4]}$ by using the method of Laplace transforms.

(4 marks)

(b) Use the result in (a) to solve the initial value problem.

(14 marks)

PART B

Q3 (a) If $P=\begin{pmatrix} -1 & 2 & -3 \\ 4 & 1 & 5 \\ 2 & -1 & 6 \end{pmatrix}$ and $Q=\begin{pmatrix} -11 & 9 & -13 \\ 14 & 0 & 7 \\ 6 & -3 & 9 \end{pmatrix}$.

(i) Find PQ and determine the inverse matrix of P.

(ii) Hence, solve the following system of linear equations.

$$-x+2y-3z=4$$
$$4x+y+5z=-5$$
$$2x-y+6z=-2$$

(9 marks)

(b) Solve the system below by using the Gauss-Jordan elimination method.

$$x+y+z=7$$
$$2x+3y-z=12$$
$$3x+2y-4z=13$$

(6 marks)

Q4 It is given that $\overrightarrow{OA}=\mathbf{i}+m\mathbf{j}+2\mathbf{k}$, $\overrightarrow{OB}=-\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ and $\overrightarrow{OC}=2\mathbf{i}+n\mathbf{j}+3\mathbf{k}$ is a placement vector for points A, B and C, where O is the origin.

(a) Find the value of m and n if $\overrightarrow{BA}\times\overrightarrow{BC}=7\mathbf{i}+2\mathbf{j}-3\mathbf{k}$.

(6 marks)

(b) Using the value of m and n from (a), determine the type of angle between vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

(2 marks)

(c) Using the answer in (a) and (b), find $\cos\angle ABC$.

(4 marks)

(d) Find the equation of a plane which passes through points A, B and C. Let $B=B_0$.

(3 marks)

Q5 (a) Given $z_1 = -1 + \sqrt{3}i$ and $z_2 = -1 - \sqrt{3}i$. Find

(i) $z_1 z_2$.

(ii) $\frac{z_1}{z_2}$.

(iii) $\frac{z_1}{\overline{z_2}}$.

(4 marks)

(b) Given $z_3 = 2 + 3i$.

(i) Write z_3 in polar form.

(i) Find $(z_3)^4$.

(ii) Find $(z_3)^{4/3}$ and write your answer in $a + ib$ form, in three decimal places.

(11 marks)

Q6 (a) Solve the following homogeneous differential equation

$$xy\frac{dy}{dx} = y^2 + x^2 e^{y/x}.$$

(7 marks)

(b) A cup of hot coffee initially at 95°C cools to 80°C in 5 minutes while sitting in a room of temperature 21°C. Determine when the temperature of the coffee will be a nice 50°C?

(8 marks)

Q7 Solve the differential equation

$$y'' + 2y' + y = e^{-x}\cos x,$$

by using the variation of parameters method.

(15 marks)

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Formulae

Laplace transform.

$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt = F(s)$	
$f(t)$	$F(s)$
k	$\dfrac{k}{s}$
$t^n,\ n = 1, 2, ..$	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$\sin at$	$\dfrac{a}{s^2+a^2}$
$\cos at$	$\dfrac{s}{s^2+a^2}$
$\sinh at$	$\dfrac{a}{s^2-a^2}$
$\cosh at$	$\dfrac{s}{s^2-a^2}$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t)\ ,\ n = 1, 2, ..$	$(-1)^n \dfrac{d^n F(s)}{ds^n}$
$y(t)$	$Y(s)$
$y'(t)$	$sY(s)-y(0)$
$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\int_0^t f(\tau)\,d\tau$	$\dfrac{F(s)}{s}$

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Differentiation And Integration Formula

Differentiation	Integration
$\frac{d}{dx} x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx} \ln x = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln\lvert x\rvert + C$
$\frac{d}{dx} e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx} \sin x = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx} \cos x = -\sin x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx} \tan x = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx} \cot x = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$
$\frac{d}{dx} \sec x = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx} \csc x = -\csc x \cot x$	$\int \csc x \cot x\, dx = -\csc x + C$
$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$
$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

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Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$;
Characteristic equation : $am^2 + bm + c = 0$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$

$f(x)$	$y_p(x)$
$P_n(x) = A_0 + A_1 x + \cdots + A_n x^n$	$x^r(B_0 + B_1 x + \cdots + B_n x^n)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

Variation of Parameters Method for $ay'' + by' + cy = f(x)$

$y(x) = uy_1 + vy_2$
$u = -\int \frac{y_2 f(x)}{aW}\,dx + A$ $\quad$ $v = \int \frac{y_1 f(x)}{aW}\,dx + B$ $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$