CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	TECHNICAL MATHEMATICS III
COURSE CODE	:	DAS 21002
PROGRAMME CODE	:	DAK
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	2 HOURS AND 30 MINUTES
INSTRUCTION	:	ANSWERS **ALL** QUESTIONS IN SECTION A AND ANSWER **THREE (3)** QUESTIONS IN SECTION B

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

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SECTION A

Q1 (a) The probability function of a discrete random variable X is given by

$$g(x) = kx^2 \text{ for } x = 1, 2, 3, 4, 5, 6$$

where k is a positive constant.

(i) Show that $k = \frac{1}{91}$.

(2 marks)

(ii) Find $P(X \geq 2)$.

(2 marks)

(iii) Find $E(X)$.

(2 marks)

(iv) Find $Var(X)$.

(4 marks)

(b) The continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} k(x^2 - 2x + 2), & 0 < x < 3 \\ 3k, & 3 < x < 4 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{9}$.

(4 marks)

(ii) Determine $P(1 < X < 3)$.

(3 marks)

(iii) Determine $P(2 < X < 5)$.

(3 marks)

Q2 (a) The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(i) Find $P(L > 127)$.

(3 marks)

(ii) Find the value of d such that $P(L < d) = 0.10$.

(3 marks)

(b) A factory produces components of which 1% are defective. The components are packed in boxes of 1000. A box is selected at random.

(i) Find the probability that the box contains at most one defective component.

(3 marks)

(ii) Find the probability that the box contains at least one defective component.

(2 marks)

(iii) Find the mean number of the defective components.

(2 marks)

(c) In a village, power cuts occur randomly at a rate of three per year.

(i) Find the probability that exactly four power cuts in one-year period.

(2 marks)

(ii) Find the mean number of power cuts in 30 month period.

(2 marks)

(iii) Find the probability that less than five power cuts in 30 month period.

(3 marks)

SECTION B

Q3 (a) Suppose $\mathbf{u} = \langle 2, -1, 3 \rangle$, $\mathbf{v} = \langle 4, 0, 2 \rangle$ and $\mathbf{w} = \langle -4, 10, -2 \rangle$.

(i) Find the magnitude of $\frac{1}{2}(\mathbf{u} + \mathbf{v}) - \frac{1}{6}\mathbf{w}$.

(4 marks)

(ii) Find $(\mathbf{u} \times \mathbf{v}) \bullet \mathbf{w}$.

(4 marks)

(iii) Find the unit vector of $\mathbf{w}$.

(2 marks)

(b) Given four points $P(1,2,3)$, $Q(1,-1,0)$, $R(2,4,7)$ and $S(0,1,1)$.

(i) Find the parametric vector equation of the line that passes through point S and parallel to vector $\overrightarrow{PQ}$.

(4 marks)

(ii) Find the equation of plane that consists of points P, Q and R.

(6 marks)

Q4 (a) If $z_1 = -4+12i$ and $z_2 = 1-3i$.

(i) Find $z_1 + 4z_2$.

(2 marks)

(ii) Express z_1 and z_2 in polar form.

(6 marks)

(iii) Determine $z_1 z_2$ by using answers in (a)(ii).

(2 marks)

(b) Given $z = (2-i)^2 + \dfrac{7-4i}{2+i} - 8$.

(i) Express z in the form of $x + iy$, where x and y are real numbers.

(5 marks)

(ii) By using answer in (b)(i) and De Moivre's Theorem, find all the second roots for z.

(5 marks)

Q5 **Table Q5** represents the times in (minutes) recorded for a group of teenagers to complete to a 5 km run.

Table Q5

Times (minutes)	Number of teenagers
16 - 20	62
21 - 25	88
26 - 30	16
31 - 35	13
36 - 40	11
41 - 45	10

(a) If x is the midpoint and f is the frequency, construct a table that contains class limit, lower boundary, x, x^2, f, cumulative frequency, $f_i x_i$, $f_i x_i^2$, Σf, $\Sigma f_i x_i$, $\Sigma f_i x_i^2$.

(8 marks)

(b) Find the mean, median, mode, variance and standard deviation for the times to complete the run.

(12 marks)

Q6 (a) A survey of the reading habits of some students revealed that, on a regular basis, 25% read quality newspaper, 45% read tabloid newspapers and 40% do not read newspaper at all.

(i) Draw a Venn diagram to represent this information.

(2 marks)

(ii) Find the probability of students who read both quality and tabloid newspaper.

(1 mark)

(b) Many educational institutions, especially higher education institutions, are considering to embrace smartphones as part of learning aids in classes as most students (in many cases all students) not only own them but also are also attached to them. A short survey was conducted to identify whether academic staffs agree with the statement or not. The two-way classification **Table Q6 (b)** below shows the responses of these students.

Table Q6 (b)

Satisfaction / **Gender**	**Agree**	**Disagree**
Male	72	25
Female	104	89

(i) Find the marginal probabilities of the events male, female, agreed and disagreed respectively.

(3 marks)

(ii) Find the probability of selecting female or disagreed respondents.
(2 marks)

(iii) Find $P(agreed \mid female)$.
(2 marks)

(c) A survey reported that 7% of all students in UTHM own a tab. 600 students were chosen randomly.

(i) Show that normal distribution can be used as an approximation to this binomial distribution. Hence, find the mean and standard deviation.
(5 marks)

(ii) By using approximating with a normal distribution, find the probability that at least 140 students own a tab.
(5 marks)

- END OF QUESTIONS -

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Table 1: Vector

$\lvert\mathbf{u}\rvert = \sqrt{a^2+b^2+c^2}$	$\hat{\mathbf{u}} = \dfrac{\mathbf{u}}{\lvert\mathbf{u}\rvert}$
$\mathbf{u} \bullet \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$	$\mathbf{u} \bullet \mathbf{v} = \lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert\cos\theta$
$\theta = \cos^{-1}\left(\dfrac{\mathbf{u}\bullet\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}\right)$	$A = \dfrac{1}{2}\lvert\mathbf{u}\times\mathbf{v}\rvert$
$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$	$x = x_0 + a_1t$ $y = y_0 + a_2t$ $z = z_0 + a_3t$
$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$	

Table 2: Complex Number

$z = a + bi$ $\bar{z} = a - bi$	$z = r(\cos\theta + i\sin\theta)$
$r = \sqrt{a^2+b^2}$	$\theta = \tan^{-1}\left(\dfrac{b}{a}\right)$
$z_1z_2 = r_1r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$	
$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)]$	
$z = re^{i\theta}$	$z^n = r^ne^{in\theta}$
$z^{\frac{1}{n}} = r^{\frac{1}{n}}e^{\left(\frac{\theta+2k\pi}{n}\right)i}$	$z^n = r^n[\cos n\theta + i\sin n\theta]$
$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\dfrac{\theta+2k\pi}{n} + i\sin\dfrac{\theta+2k\pi}{n}\right)$	

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Table 3: Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	$P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$

Table 4: Descriptive Statistics

$\mu = \dfrac{\sum_{i=1}^{n} x_i}{N}$	$\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum f}$
$\sigma^2 = \dfrac{\sum_{i=1}^{n} (x_i - \mu)^2}{N}$	$s^2 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$
$s^2 = \dfrac{1}{\Sigma f - 1}\left[\sum_{i=1}^{n} f_i x_i^2 - \dfrac{(\Sigma f_i x_i)^2}{\Sigma f}\right]$	
$M = L_m + C \times \left(\dfrac{\frac{n}{2} + F_{m-1}}{f_m}\right)$	$M_0 = L + C \times \left(\dfrac{d_b}{d_b + d_a}\right)$

Table 5: Probability Distribution

Binomial $X \sim B(n,p) = \binom{n}{r} p^r (1-p)^{n-r}$ for $n = 0,1,\ldots,n$
Poisson $X \sim P_O(\mu) = \dfrac{e^{-\mu}\mu^r}{r!}$ for $\mu = 0,1,2\ldots$
Normal $X \sim N(\mu, \sigma^2)$, $f(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\left[\frac{(x-\mu)^2}{2\sigma^2}\right]}$
Standard Normal $Z \sim N(0,1)$, $f(z) = \dfrac{1}{\sqrt{2\pi}} e^{-\left[\frac{z^2}{2}\right]}$, $z = \dfrac{x-\mu}{\sigma}$

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Table 6: Random Variables

$\sum_{i=-\infty}^{\infty} p(x_i) = 1$	$\int_{-\infty}^{\infty} f(x)\ dx = 1$
$E(X) = \sum_{\forall x} xp(x)$	$E(X) = \int_{-\infty}^{\infty} xp(x)\ dx$
$Var(X) = E(X^2) - [E(X)]^2$	