Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ALGEBRA
COURSE CODE	:	DAS 10103/ DAM 10303/ DAE 13003
PROGRAMME CODE	:	DAU / DAM / DAE
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN SECTION A AND **THREE (3)** QUESTIONS IN SECTION B

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

SECTION A

Q1 (a) Given $\boldsymbol{u} = 7\boldsymbol{i} - \boldsymbol{k}$, $\boldsymbol{v} = 3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{w} = 6\boldsymbol{i} + \boldsymbol{j} - 5\boldsymbol{k}$. Find

(i) $4\boldsymbol{u} - 3\boldsymbol{v} + \boldsymbol{w}$

(2 marks)

(ii) $\boldsymbol{v} \bullet (3\boldsymbol{u} \times 2\boldsymbol{w})$.

(6 marks)

(b) Find the equation of the line that passes through points $A(1, -3, 7)$ and $B(-1, -2, 5)$.

(4 marks)

(c) Given three points $P(-3, 1, 4)$, $Q(-2, 0, 3)$ and $R(0, -1, 5)$. Find

(i) the normal vector, $\boldsymbol{n}$ where $\boldsymbol{n} = \boldsymbol{PQ} \times \boldsymbol{PR}$.

(6 marks)

(ii) the equation of the plane with points P, Q and R on it, by using point P and $\boldsymbol{n}$ in **Q1** (c)(i).

(2 marks)

Q2 (a) The complex number z_1 and z_2 are given by :

$$z_1 = -3 + 2i \quad \text{and} \quad z_2 = 2 - 1i$$

(i) Find $\frac{z_1}{z_2}$ in the form of $a + bi$ where a and b are real numbers.

(4 marks)

(ii) Express $z_1 z_2$ in polar form.

(6 marks)

(b) Determine $\left(4 + \sqrt{3}i\right)^5$ in the form of $a + bi$ using De Moivre's Theorem.

(5 marks)

(c) Find all the 3rd root of $z = -27i$ by using De Moivre's Theorem.

(5 marks)

SECTION B

Q3 (a) (i) Simplify $(5x^3y^{10})(2x^4y^2z^{-3})^3$ and write the answer in positive exponent.

(2 marks)

(ii) Given $27^x - \frac{9}{3^{-x}} = 0$. Find x.

(4 marks)

(b) Simplify and give the answer in simplest form.

(i) $\frac{\sqrt{7}+9}{\sqrt{7}-1}$

(3 marks)

(ii) $\frac{\sqrt[3]{x^2y^3}\sqrt[3]{125x^3}}{\sqrt[3]{8x^3y^4}}$

(4 marks)

(c) (i) Given $\log_2(\sqrt{256})^x = \log_{32} 16$. Find x.

(4 marks)

(ii) Given $\log_5 \sqrt[3]{25} = x$. Find x without using calculator.

(3 marks)

Q4 (a) (i) Solve the quadratic equation $2s^2 + 5s = 3$, by using completing the square.

(5 marks)

(ii) Use the quadratic formula to solve $x^2 = 4x + 8$. Leave your answer in the form of $a \pm b\sqrt{c}$.

(5 marks)

(b) Determine the partial fraction decomposition of $\frac{4x^2}{(x-1)(x-2)^2}$.

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(5 marks)

(c) Given $f(x) = x^3 - 10x + 10$. If $f(x) = 0$, by secant method, find its roots between the interval of $[1, 2]$. Iterate until $|f(x_i)| < \varepsilon = 0.005$.

(5 marks)

Q5 (a) Find the sum of $\sum_{k=1}^{19}\left(4k-5k^3+\frac{k^2}{5}\right)$

(5 marks)

(b) The 5^{th} and 11^{th} term of an arithmetic sequence are – 7 and 35 respectively.

(i) Find the first term and the common difference.

(5 marks)

(ii) Find the sum of the first 45 terms.

(2 marks)

(c) A geometric sequence is defined as $9, 6, 4, \ldots$

(i) Find the value of common ratio, r.

(2 marks)

(ii) Calculate the 15^{th} term, a_{15}.

(2 marks)

(iii) State whether this series converges or diverges. If it is converges, evaluate its summation S_∞.

(4 marks)

Q6 (a) Without using calculator, find the exact value in surd form of :

(i) $2\sec 30^0 + 3\csc 60^0$

(3 marks)

(ii) $\cos 75°$

(3 marks)

(b) Solve the equation, for $0 \le \theta \le 2\pi$.

(i) $\tan\theta = -0.5543$

(3 marks)

(ii) $\sin^2\theta + ½\cos\theta = 1$

(4 marks)

(c) Given $5\sin\theta + 12\cos\theta = r\sin(\theta+\alpha)$ and $0° \le \theta \le 360°$.

(i) Find r and α.

(3 marks)

(ii) Thus, find the value of θ if $5\sin\theta + 12\cos\theta = 6$.

(4 marks)

Q7 (a) Given $A = 2\begin{bmatrix} 1 & x & 5 \\ y & 4 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -x \\ 0 & 3 \\ z & z \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 5 \\ 6 & -3 \end{bmatrix}$.

(i) Solve x, y and z if AB = C.

(5 marks)

(ii) Find $(AB)^T$.

(4 marks)

(b) Given :

$$2x + y + z = 3$$
$$-3x - 2y = -7$$
$$3x + y - z = 6$$

(i) Write the matrix equation AX = B of the above system of equation.

(1 mark)

(ii) Find the determinant of matrix A.

(2 marks)

(iii) Solve the above system for x, y and z by using Gauss-Jordan elimination method. Do this following operation in order: $R_2 + R_3$, $R_3 - R_1$, $R_3 \Leftrightarrow R_1$, $R_3 - 2R_1$, $-R_2$, $R_3 - R_2$, $\frac{R_3}{4}$, $R_1 + 2R_3$, $R_2 - R_3$.

(8 marks)

- END OF QUESTION -

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Formulae

Vector

$$\boldsymbol{a} \bullet \boldsymbol{b} = x_1x_2 + y_1y_2 + z_1z_2, \quad \boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$$x = x_0 + a_1 t, \quad y = y_0 + a_2 t, \quad z = z_0 + a_3 t \text{ and } \frac{x - x_o}{a_1} = \frac{y - y_o}{a_2} = \frac{z - z_o}{a_3}$$

$$A(x - x_o) + B(y - y_o) + C(z - z_o) = 0$$

Complex number

$$r = \sqrt{x^2 + y^2} \qquad \tan\theta = \frac{y}{x}$$

If $z = r(\cos\theta + i\sin\theta)$ then $z^n = r^n(\cos n\theta + i\sin n\theta)$

If $z = r(\cos\theta + i\sin\theta)$ then $z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{(\theta + 2k\pi)}{n} + i\sin\frac{(\theta + 2k\pi)}{n}\right)$

Exponent, Logarithm and Radical

$$\log_a x = \frac{\log_a x}{\log_a b}$$

Polynomials

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c, \qquad x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

Sequence and Series

$$\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$u_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d], \qquad S_n = \frac{n}{2}(a + u_n), \qquad u_n = S_n - S_{n-1}$$

$$u_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r - 1}, r > 1 \text{ or } S_n = \frac{a(1 - r^n)}{1 - r}, r < 1, \quad S_\infty = \frac{a}{1 - r}.$$

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Trigonometry

Angle θ	sin θ	cos θ	tan θ
30°	1/2	$\sqrt{3}/2$	$1/\sqrt{3}$
45°	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\sqrt{3}/2$	1/2	$\sqrt{3}$

$\sin^2 x + \cos^2 x = 1,\qquad \tan^2 x + 1 = \sec^2 x,\qquad 1 + \cot^2 x = \csc^2 x$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta,\qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\sin 2\theta = 2\sin\theta\cos\theta,\qquad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$a\sin\theta + b\cos\theta = r\sin(\theta + \alpha),\ r = \sqrt{a^2 + b^2}$ and $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix},\quad |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$