Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAC 20103 / DAS 20403
PROGRAMME CODE	:	DAA
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN SECTION A AND **THREE (3)** QUESTIONS IN SECTION B

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

SECTION A

Q1 (a) Find the inverse of the following Laplace transform:

(i) $\dfrac{35}{s^2+5^2}-\dfrac{2}{s+3}+\dfrac{4}{s}$.

(3 marks)

(ii) $\dfrac{21}{(s+1)^2-49}-\dfrac{s}{(s-2)^2+4^2}$.

(3 marks)

(iii) $\dfrac{s+6}{s^2-2s+17}-\dfrac{s+3}{s^2+4s+13}$.

(6 marks)

(b) (i) Express $\dfrac{s^2+14s+4}{s^3-12s+16}$ as partial fraction.

(5 marks)

(ii) Hence, find the inverse Laplace of the partial fraction from **Q1 (b) (i)**.

(3 marks)

Q2 Solve the given initial value problem below by using Laplace transform.

(a) $y''-3y'+2y=e^t$; $y(0)=1,\quad y'(0)=0$.

(9 marks)

(b) $2y''-5y'-7y=2t$; $y(0)=0,\quad y'(0)=0$.

(11 marks)

SECTION B

Q3 (a) Given the first order linear differential equation $\sqrt{(1+x^2)}\ dy + xy\ dx = 0$. Solve by using the method of separable equation.

(5 marks)

(b) Given the first order linear differential equation $(x^2 - xy)\frac{dy}{dx} + y^2 = 0$. Solve by using the method of homogeneous equation.

(7 marks)

(c) Given the first order linear differential equation $3x - 2y = (2x - 3y)\frac{dy}{dx}; y(2) = 2$. Solve by using the method of exact equation.

(8 marks)

Q4 (a) According to Newton's law of cooling, the rate at which a body cooled is proportional to the difference between the temperature of the body and that of the surrounding medium. Let θ represent the temperature in $°C$ of a corpse in a mortuary whose the temperature is kept at a constant $2\,°C$ If the corpse cools from $37\,°C$ to $20\,°C$ in 20 minutes, find the time taken by the corpse to drop to $10\,°C$ and determine the temperature of the corpse after 2 hours.

(10 marks)

(b) A certain city had a population of 25000 in year 1990 and a population of 30000 in year 2000. Assume that its population will continue to grow exponentially at a constant rate. Calculate the population of that city in the year 2030 and determine at which year the population would reach 1.2 million.

(10 marks)

Q5 (a) Find the Laplace transform of the following functions:

(i) $f(t) = \sin 2t + \cos 3t$.

(3 marks)

(ii) $f(t) = 5e^{3t}\cos 5t$.

(4 marks)

(iii) $f(t) = 5e^{t} - t\sin 3t$.

(4 marks)

(iv) $f(t) = 3e^{t} + 2e^{-t}$.

(3 marks)

(b) Find the Laplace transform for $e^{5t}\cosh 2t + t^4 e^{-3t}$ by using the First Shift theorem.

(6 marks)

Q6 (a) Using undetermined coefficients method, find the general solution of:

$$y''+2y'+y=3e^{-x}$$

(10 marks)

(b) Using the variation of parameter method, solve the following second order differential equation:

$$y''+9y=3\sec(3x)$$

(10 marks)

Q7 Solve the following initial and boundary problems by using Laplace transform:

(a) $y''+2y'-3y=t$ where $y(0)=2, y'(0)=1.$

(11 marks)

(b) $y'+y=\sin t$ where $y\left(\frac{\pi}{2}\right)=\frac{1}{2}.$

(9 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2019/2020 PROGRAMME CODE: DAA
COURSE NAME : ENGINEERING MATHEMATICS II COURSE CODE : DAC 20103/ DAS 20403

Formulae

Table 1: Laplace and Inverse Laplace Transforms

$L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$	
$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n, n = 1,2,\ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
First Shift Theorem	
$e^{at}f(t)$	$F(s-a)$
Multiply with t^n	
$t^n f(t), n = 1,2,\ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$
Convolution Property:	
If $F(s) = G(s)H(s)$, then $L^{-1}G(s) = g(t)$ and $L^{-1}H(s) = h(t)$. $f(t) = L^{-1}F(s) = L^{-1}[G(s)H(s)] = \int_0^t g(\tau)h(t-\tau)d\tau$ or $\int_0^t h(\tau)g(t-\tau)d\tau$	
Initial Value Problem	
$L\{y(t)\} = Y(s)$	
$L\{y'(t)\} = sY(s) - y(0)$	
$L\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$	

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2019/2020 PROGRAMME CODE: DAA
COURSE NAME : ENGINEERING MATHEMATICS II COURSE CODE : DAC 20103/ DAS 20403

Table 2: Differentiation

$\frac{d}{dx}[c]=0$	$\frac{d}{dx}[\sin u]=\cos u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos u]=-\sin u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[cu]=c\,\frac{du}{dx}$	$\frac{d}{dx}[\ln\lvert u\rvert]=\frac{1}{u}\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[u\pm v]=\frac{du}{dx}\pm\frac{dv}{dx}$	$\frac{d}{dx}\left[e^u\right]=e^u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[uv]=u\frac{dv}{dx}+v\frac{du}{dx}$	$\frac{d}{dx}[a^u]=a^u\ln a\left(\frac{du}{dx}\right)$
$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}[\tan u]=\sec^2 u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[\log_a u]=\frac{1}{u}\log_b e\left(\frac{du}{dx}\right)$	$\frac{d}{dx}[\sec u]=\sec u\tan u\left(\frac{du}{dx}\right)$

Table 3: Integration

$\int x^n dx=\frac{x^{n+1}}{n+1}+C$	$\int\cos ax\ dx=\frac{1}{a}\sin ax+C$
$\int\frac{1}{x}dx=\ln\lvert x\rvert+C$	$\int\tan ax\ dx=\frac{1}{a}\ln\lvert\sec ax\rvert+C$
$\int\frac{1}{a+bx}dx=\frac{1}{b}\ln\lvert a+bx\rvert+C$	$\int udv\ dx=uv-\int vdu$
$\int\sin ax\ dx=-\frac{1}{a}\cos ax+C$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+C$

FINAL EXAMINATION

SEMESTER / SESSION	: SEM I / 2019/2020	PROGRAMME CODE: DAA	
COURSE NAME	: ENGINEERING MATHEMATICS II	COURSE CODE	: DAC 20103/ DAS 20403

Table 4: Characteristic Equation and General Solution

Homogeneous Differential Equation: $ay'' + by' + cy = 0$

Characteristics Equation: $am^2 + bm + c = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case	Roots of Characteristics Equation	General Solution
1	Real and Distinct: $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	Real and Equal: $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	Complex Roots: $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 5: Particular Solution of Nonhomogeneous Equation

Nonhomogeneous Differential Equation: $ay'' + by' + cy = 0$

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \ldots + A_1 x + A_0$	$x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)$
Ce^{ax}	$x^r\left(Pe^{ax}\right)$
$C\cos\beta x$ or $C\sin\beta x$	$x^r\left(P\cos\beta x + Q\sin\beta x\right)$
$P_n(x)e^{ax}$	$x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)e^{ax}$
$P_n(x)\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$ or	$x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)\cos\beta x$ + $x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)\sin\beta x$

Notes: r is the smallest non negative integer to ensure no alike term between $y_p(x)$ and $y_h(x)$.