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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ALGEBRA
COURSE CODE	:	DAC 11303
PROGRAMME CODE	:	DAA
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN SECTION A AND **TWO (2)** QUESTIONS IN SECTION B

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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SECTION A

Q1 (a) Identify the root of the $2x - \sin x - \frac{1}{2} = 0$ in the interval $[1,2]$ using secant method. Iterate until $|f(x_i)| < \varepsilon = 0.005$. Show your calculation in four decimal places.

(10 marks)

(b) Express $\dfrac{Z^3 + 6Z^2 + 17Z + 16}{Z^2 + 4Z + 3}$ in the form of partial fraction.

(10 marks)

Q2 (a) Given $\boldsymbol{p} = 2\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$, $\boldsymbol{q} = \boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$, $\boldsymbol{r} = 5\boldsymbol{i} - 2\boldsymbol{k}$ and $\boldsymbol{s} = a\boldsymbol{i} - \boldsymbol{j} - 4b\boldsymbol{k}$. Find

(i) $2(\boldsymbol{q} - 3\boldsymbol{r})$

(2 marks)

(ii) $|\boldsymbol{p} + 3\boldsymbol{q} + 4\boldsymbol{r}|$.

(4 marks)

(iii) The value of a and b if $4\boldsymbol{p} - 3\boldsymbol{q} + \boldsymbol{s} = \boldsymbol{p}.\boldsymbol{q}$

(4 marks)

(b) If $\boldsymbol{u} = 4\boldsymbol{i} + 5\boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{v} = -\boldsymbol{i} + 3\boldsymbol{j} - 5\boldsymbol{k}$, and $\boldsymbol{w} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$, calculate $\boldsymbol{u}(\boldsymbol{v} \times \boldsymbol{w})$

(4 marks)

(c) Given three points $D(1,3,-1), E(-1,-1,2)$ and $F(-2,0,4)$. Determine the equation of the plane passing through all the points.

(6 marks)

Q3 (a) The complex number z_1 and z_2 are given by

$$z_1 = p + 4i \quad \text{and} \quad z_2 = 1 - 4i$$

where p is an integer.

(i) Compute $\frac{z_1}{z_2}$ in the form of $a + bi$ where a and b are real. Give your answer in its simplest form in terms of p.

(4 marks)

(ii) Given that $\left|\frac{z_1}{z_2}\right| = 14$. Find the possible values of p.

(4 marks)

(iii) Express $z_1 z_2$ in polar form. (Hint: Use p positive number).

(6 marks)

(b) Calculate $(4 + 4i)^5$ in the form of $a + bi$ using De Moivre's Theorem.

(6 marks)

SECTION B

Q4 (a) Solve

(i) $3^{2z} - 6 \cdot 3^{z} = 27$

(4 marks)

(ii) $3^{m+3} - 6 \cdot 3^{m+1} = 81$

(4 marks)

(b) Simplify $10\sqrt{3} + 7\sqrt{3} - 4\sqrt{3}$ into the simplest form.

(3 marks)

(c) Rationalize the denominator in $\dfrac{6}{3\sqrt{5} + 2\sqrt{7}}$

(3 marks)

(d) (i) Interpret $2\log_k 5 + \dfrac{1}{2}\log_k 9 - \log_k 3 = \log_k x$

(4 marks)

(ii) Solve $\log_e \dfrac{x^2}{5} = \log_e x^2 - \log_e 5$

(2 marks)

Q5 (a) Simplify the sum of $\sum_{k=1}^{17}\left(\dfrac{k^3}{2} - 3k^2 + k\right)$.

6 marks)

(b) Given the sequence $-13, -7, -1, \ldots$

(i) Determine whether the sequence is an arithmetic sequence or geometric sequence. Hence, find T_n.

(4 marks)

(ii) Identify $S_{19} - S_6$.

(2 marks)

(c) (i) Determine the number of terms in the Geometric Sequence 4, 3.6, 3.24, … needed so that the sum exceeds 35.

(4 marks)

(ii) The sum to infinity of a Geometric Sequence is twice the sum of the first two terms. Classify possible values of the common ratio.

(4 marks)

Q6 (a) If $\sin\theta = \frac{5}{13}$ and θ lies in quadrant II, compute the value of

(i) $\tan 2\theta$

(2 marks)

(ii) $\sin 2\theta$

(2 marks)

(b) Analyze the exact value of $\sin(60°)$ using half-angle formula.

(6 marks)

(c) (i) By using sum and difference identities, show

$$\frac{1+\tan x}{1-\tan x} = \tan(45^0 + x)$$

(4 marks)

(ii) Solve the trigonometric equation

$$9\tan\theta + \tan^2\theta = 5\sec^2\theta - 3; \quad 0° \le \theta \le 360°.$$

(6 marks)

Q7 (a) Given

$$a + 3b + 2c = 2$$
$$2a + 7b + 7c = -1$$
$$2a + 5b + 2c = 7$$

(i) Express the matrix equation AX = B of the system equation.

(1 marks)

(ii) Show the determinant of matrix A

(3 marks)

(iii) Identify inverse for matrix A

(6 marks)

(iv) Use inversion to solve a, b and c

(4 marks)

(b) Solve the below system for *x*, *y* and *z* by using Gauss-Jordan elimination method.

$$2y + z = 4$$
$$x + y + 2z = 6$$
$$2x + y + z = 7$$

Do this following operation in order:

$R_1 \leftrightarrow R_2$,

$R_3 + (-2R_1)$,

$R_1 + (-\frac{1}{2}R_2)$,

$R_3 + (\frac{1}{2}R_2)$,

$R_1 + (\frac{3}{5}R_3)$,

$R_2 + (\frac{2}{5}R_3)$,

$\frac{1}{2}R_2$,

$\frac{R_3}{(-\frac{5}{2})}$

(6 marks)

END OF QUESTION

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Formulae

Polynomials

$$\log_a x = \frac{\log_a x}{\log_a b}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c, \quad x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

Sequence and Series

$$\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$u_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d], \quad S_n = \frac{n}{2}(a + u_n)$$

$$u_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r-1}, r > 1 \text{ or } S_n = \frac{a(1-r^n)}{1-r}, r < 1, \quad S_\infty = \frac{a}{1-r}.$$

$$u_n = S_n - S_{n-1}$$

$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots\ldots$$

Trigonometry

$$\sin^2 x + \cos^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}, \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}, \quad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

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$a\sin\theta + b\cos\theta = r\sin(\theta+\alpha) = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = (r\cos\alpha)\sin\theta + (r\sin\alpha)\cos\theta$ and

$a = r\cos\alpha$ and $b = r\sin\alpha$

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}}{a_{11}}, \quad x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}, \quad x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$$

Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$Adj(A) = (c_{ij})^T \qquad A^{-1} = \frac{1}{|A|}Adj(A)$$

Vector

$$\cos\theta = \frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \quad \text{or} \quad \mathbf{a}\bullet\mathbf{b} = x_1x_2 + y_1y_2 + z_1z_2, \quad \mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$|\mathbf{a}| = \sqrt{x^2+y^2+z^2}$, $x = x_0 + a_1 t$, $y = y_0 + a_2 t$, $z = z_0 + a_3 t$ and $\dfrac{x-x_o}{a_1} = \dfrac{y-y_o}{a_2} = \dfrac{z-z_o}{a_3}$

$A(x-x_o) + B(y-y_o) + C(z-z_o) = 0$

Complex number

$r = \sqrt{x^2+y^2}$ $\qquad \tan\theta = \dfrac{y}{x}$

If $z = re^{i\theta}$, then $z^n = r^n e^{in\theta}$

If $z = re^{i\theta}$, then $z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{\left(\frac{\theta+2k\pi}{n}\right)i}$.

If $z = r(\cos\theta + i\sin\theta)$ then $z^n = r^n(\cos n\theta + i\sin n\theta)$

If $z = r(\cos\theta + i\sin\theta)$ then $z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\dfrac{(\theta+2k\pi)}{n} + i\sin\dfrac{(\theta+2k\pi)}{n}\right)$