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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

**FINAL EXAMINATION
SEMESTER II
SESSION 2017/2018**

COURSE NAME : TECHNICAL MATHEMATICS II
COURSE CODE : DAS 11103
PROGRAMME CODE : DAK / DAJ
EXAMINATION DATE : JUNE / JULY 2018
DURATION : 3 HOURS
INSTRUCTION : ANSWER ALL QUESTIONS IN
PART A AND **THREE (3)**
QUESTIONS IN PART B

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THIS QUESTION PAPER CONSISTS OF **9 (NINE)** PAGES

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PART A

- Q1** (a) **Figure Q1(a)** shows that the region **T** bounded by $x = y^2 - y - 6$ and $x = 2y + 4$.
- (i) Find the coordinate of points **P** and **Q**. (4 marks)
- (ii) Find the area of the shaded region, **T**. (4 marks)
- (b) By using the cylindrical shells method, find the volume of the region bounded by the curves $x = (y - 2)^2$ and $x = y$ when it is revolved about the x -axis. (5 marks)
- (c) Find the arc length of the curve, $y = \frac{2}{3}(x - 1)^{\frac{3}{2}}$ when $1 \leq x \leq 2$. (7 marks)
- Q2** (a) Evaluate
- (i) $\int 5x^2 - 8x + 5 \, dx$. (2 marks)
- (ii) $\int_{-1}^2 t^2 + 3t^3 \, dt$. (3 marks)
- (b) By using substitution method, find $\int_0^1 x\sqrt{1+x^2} \, dx$. (7 marks)
- (c) Solve $\int \sin x \ln(\cos x) \, dx$ by using integration by parts. (8 marks)

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PART B

Q3 (a) By using Trapezoidal rule, solve the following integral with $h = 0.6$.

Write the answer to 4 decimal places.

$$\int_{-2}^4 \sin(x^2 + 2) dx.$$

(6 marks)

(b) By using Simpson's rule, solve the following integral with $n = 12$. Write the answer to 4 decimal places.

$$\int_0^4 \sqrt[3]{x^4 + 6} dx.$$

(7 marks)

(c) Solve the following integral using partial fraction.

$$\int_3^4 \frac{4x+1}{x^2-x-2} dx.$$

(7 marks)

Q4 (a) Given the function of $y = x^4 - 8x^2$.

(i) Find the extrema and the inflection points.

(10 marks)

(ii) Hence, sketch the graph.

(3 marks)

(b) A spherical balloon is being inflated in such a way that its volume is increasing at a constant rate of $180\text{cm}^3\text{s}^{-1}$. At time t seconds, the radius of the balloon is r cm. Find

(i) $\frac{dr}{dt}$ when $r=30$.

(4 marks)

(ii) the rate of increase of the surface area of the balloon when its radius is 30 cm. [Hint: $A = 4\pi r^2$].

(3 marks)

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Q5 (a) Find $\frac{dy}{dx}$ of the following functions:

(i) $y = \sin(2x) + \frac{3}{\sqrt{x}} + \frac{2}{\sqrt{x^3}}$.

(3 marks)

(ii) $y = 2e^x(3 + 4x - x^2)$.

(4 marks)

(b) By using chain rule, differentiate $g(t) = -t^{-5} + \left(\frac{4}{t^3} - 7t\right)^3$.

(7 marks)

(c) By using implicit differentiation, find $\frac{dy}{dx}$ of the function

$y^5 + x^2y^3 = 1 + x^4y$ when $x = 1$ and $y = 2$.

(6 marks)

Q6 (a) The graph of $f(x)$ is shown in **Figure Q6(a)**. Using the properties of continuity, determine whether the function is continuous when

(i) $x = 1$.

(4 marks)

(ii) $x = 4.5$.

(4 marks)

(b) Given that $f(x) = \begin{cases} x^3 & , \quad x < -1 \\ ax + b & , \quad -1 \leq x < 1. \\ x^2 + 2 & , \quad x \geq 1 \end{cases}$. If $f(x)$ is a continuous

function, find the values of a and b .

(5 marks)

(c) Find

(i) $\lim_{x \rightarrow -2} \frac{3x^2 - 12}{x + 2}$.

(3 marks)

(ii) $\lim_{x \rightarrow 0} \frac{x(1 - 2x)}{1 - \sqrt{x+1}}$.

(4 marks)

Q7 (a) The function $f(x)$ is given by

$$f(x) = \begin{cases} 4 & , \quad x \leq -2 \\ x^2 & , \quad -2 < x < 2. \\ x+2 & , \quad x \geq 2 \end{cases}$$

(i) Sketch the graph of $f(x)$.

(4 marks)

(ii) State the domain and range of $f(x)$.

(2 marks)

(iii) Find the value of $f(x)$ when $x = -4$, $x = 1$ and $x = 6$.

(3 marks)

(b) The functions $h(x)$ and $g(x)$ are defined by

$$g(x) = \frac{1}{x+3} \text{ and } h(x) = \frac{2}{x-4}.$$

(i) Find $g^{-1}(x)$.

(3 marks)

(ii) Find and simplify the composite function $g \circ h$.

(4 marks)

(iii) Find an expression for $(g \circ h)^{-1}(x)$.

(4 marks)

–END OF QUESTIONS–

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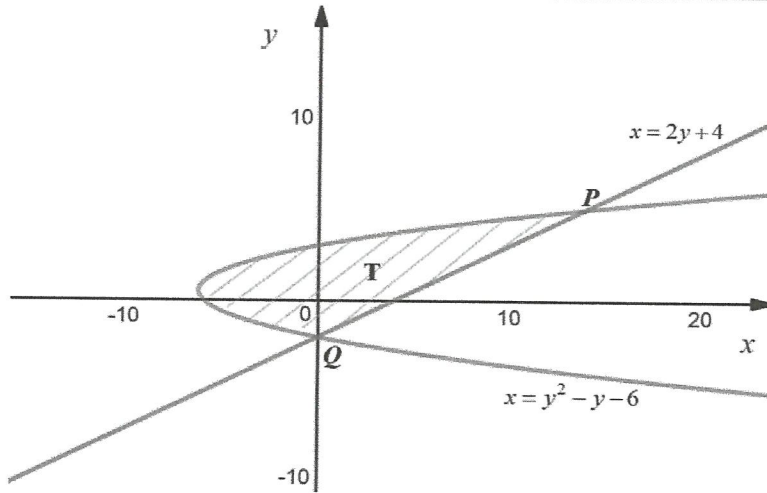


Figure Q1(a)

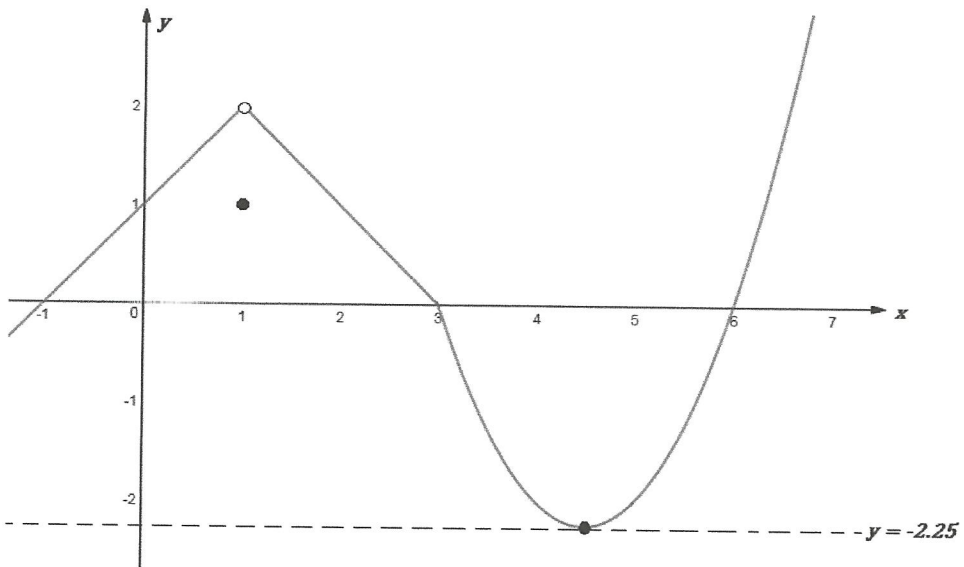


Figure Q6(a)

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Table 1: Differentiation

General Rule	
$\frac{d}{dx}(ax^n) = nax^{n-1}$	$\frac{d}{dx}(\sin x) = \cos x$
$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(\sin u) = \cos u \left(\frac{du}{dx} \right)$
$\frac{d}{dx}(e^{ax+b}) = ae^{ax+b}$	$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(a^x) = a^x \ln a$	$\frac{d}{dx}(\tan x) = \sec^2 x$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$
$\frac{d}{dx}(\ln u) = \frac{1}{u} \left(\frac{du}{dx} \right)$	$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\log_b x) = \frac{1}{x} \log_b e$	$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
Product and Quotient Rule	
$\frac{d}{dx}(uv) = uv' + vu'$	
$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{vu' - uv'}{v^2}$	
Chain Rule	Parametric Differentiation
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$	$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$
$\frac{dy}{dx} = \frac{dy}{dm} \cdot \frac{dm}{dn} \cdot \frac{dn}{dx}$	

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Table 2: Integration

General Rules:

$$\int k dx = kx + C$$

$$\int e^x dx = e^x + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$\int \frac{1}{x} dx = \ln x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| + C$$

$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$$

Definite Integral:

$$\int_a^b f(x) dx = F(b) - F(a)$$

Integration by Parts:

$$\int u dv = uv - \int v du$$

Partial Fraction:

$$\frac{a}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

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Table 3: Integration

Trapezoidal Rule

$$\int_a^b f(x) dx \approx \frac{h}{2} \left[f(a) + f(b) + 2 \sum_{i=1}^{n-1} f(a+ih) \right]$$

Simpson's Rule

$$\int_a^b f(x) dx \approx \frac{h}{3} \left[f(a) + f(b) + 4 \sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f(a+ih) + 2 \sum_{\substack{i=1 \\ i \text{ even}}}^{n-1} f(a+ih) \right]$$

Area

$$A = \int_a^b [f(x) - g(x)] dx$$

$$A = \int_c^d [f(y) - g(y)] dy$$

Volume in Cylindrical Shells

$$V = 2\pi \int_a^b x[f(x) - g(x)] dx$$

$$V = 2\pi \int_c^d y[f(y) - g(y)] dy$$

Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

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