UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2017/2018

COURSE NAME	:	CALCULUS
COURSE CODE	:	DAS 20803
PROGRAMME CODE	:	DAU
EXAMINATION DATE	:	JUNE / JULY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A ANSWERS **ALL** QUESTIONS SECTION B ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

SECTION A

Q1 (a) Find the following integrals.

(i) $\int \frac{x^5 + 2x^3 - 1}{x^4}\, dx.$

(3 marks)

(ii) $\int \left(\sin 2x + \cos 4x - e^{-2x}\right) dx.$

(3 marks)

(b) Solve the following integrals using the given technique.

(i) $\int \frac{3x^2 + 1}{(x^3 + x + 10)^2}\, dx$; [substitution] .

(3 marks)

(ii) $\int xe^{2x}\, dx$; [by part] .

(4 marks)

(iii) $\int \frac{3x + 2}{x^2 + 3x + 2}\, dx$; [partial fraction].

(4 marks)

(c) Evaluate $\int_1^4 \left(x + \frac{1}{x}\right) dx$.

(3 marks)

Q2 (a) Determine the area of the region enclosed by $x = \frac{1}{2}y^2 - 3$ and $x = y + 1$ (see **Figure Q2(a)**).

(6 marks)

(b) Determine the volume of the solid obtained by rotating about the x – axis, region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$ and the x – axis (see **Figure Q2(b)**).

(7 marks)

(c) Using Simpson's rule with h = 0.250, solve $\int_0^{1.5} \sqrt{\frac{x}{1+x}}\, dx$. Write the answer to three (3) decimal places.

(7 marks)

SECTION B

Q3 (a) Function $f(x)$ is given by

$$f(x)=\begin{cases} x^2 & , \quad x<2 \\ 4 & , \quad 2\leq x<3 \\ 10-2x & , \quad x\geq 3 \end{cases}.$$

(i) Sketch the graph of $f(x)$.

(3 marks)

(ii) Write the domain and range of $f(x)$.

(2 marks)

(iii) Find the value of $f(1)$ and $f(5)$.

(2 marks)

(b) Given $f(x)=x+2$, $g(x)=\frac{x}{3}-1$ and $h(x)=\frac{x}{2}$. Find

(i) $f\circ g$.

(2 marks)

(ii) $f\circ(g\circ h)$.

(2 marks)

(c) If $g(x)=\dfrac{3-2x}{k}$.

(i) Find $g^{-1}(x)$.

(3 marks)

(ii) If $g^{-1}(1)=4$, find k.

(3 marks)

(d) Determine the value of a so that

$$f(x)=\begin{cases} x^2-1, & x<3 \\ 2ax & x\geq 3 \end{cases},$$

is continuous at $x=3$.

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(3 marks)

Q4 (a) Evaluate the following limits using L'Hopital's Rule only.

(i) $\lim_{x \to \infty} \dfrac{x^3 + 3}{2x^3 + 4x + 1}$.

(5 marks)

(ii) $\lim_{x \to 1} \dfrac{x^2 + 12x - 13}{x - 1}$.

(3 marks)

(iii) $\lim_{x \to 0} \dfrac{1 - \cos x}{x^2}$.

(3 marks)

(b) Given a curve of $y(x) = 2x^3 - 5x^2 - 4x + 2$. Find the

(i) critical value of x.

(4 marks)

(ii) maximum and minimum points.

(5 marks)

Q5 (a) Find $\dfrac{dy}{dx}$ of the given equation.

(i) $y = 2\sqrt{x} + 4x^2 - \dfrac{5}{x^4}$.

(3 marks)

(ii) $y = 5x \sin 2x$.

(3 marks)

(i) $y = \ln(3x^2 + 2)$.

(3 marks)

(ii) $y = \dfrac{4x^2 - 5x}{x + 3}$.

(4 marks)

(b) Using implicit differentiation find $\dfrac{dy}{dx}$ of $x^2 + xy - 2y^3 = 25$.

(4 marks)

(c) Given function, $y = 3x^3 + 9x$. Find y'' and y'''.

(3 marks)

Q6 Evaluate the following integrals using the stated method.

(a) $\int x^2 e^{2x}\, dx$; [tabular] .

(5 marks)

(b) $\int x^2 \ln x\, dx$; [by part] .

(5 marks)

(c) $\int \frac{12x^2+16}{x^3+4x}\, dx$; [substitution, $u = x^3 + 4x$] .

(5 marks)

(d) $\int_4^6 \frac{x-5}{(x-2)(x-3)}\, dx$; [partial fraction] .

(5 marks)

Q7 (a) Determine the arc length of $x = \frac{2}{3}(y-1)^{\frac{3}{2}}$ between $1 \le y \le 4$.

(6 marks)

(b) Using Trapezoidal rule, find the value for $\int_1^4 \frac{1}{\sqrt{x}+3}\, dx$ with $h = 0.50$. Write the answer to three (3) decimal places.

(8 marks)

(c) Determine whether the following integrations are improper or proper integral. Give your reason.

(i) $\int_{-1}^{2} \frac{3}{x^2-1}\, dx$.

(2 marks)

(ii) $\int_{-\infty}^{\infty} 2e^x\, dx$.

(2 marks)

(iii) $\int_2^{\infty} \frac{2}{x^2-x}\, dx$.

(2 marks)

– END OF QUESTIONS –

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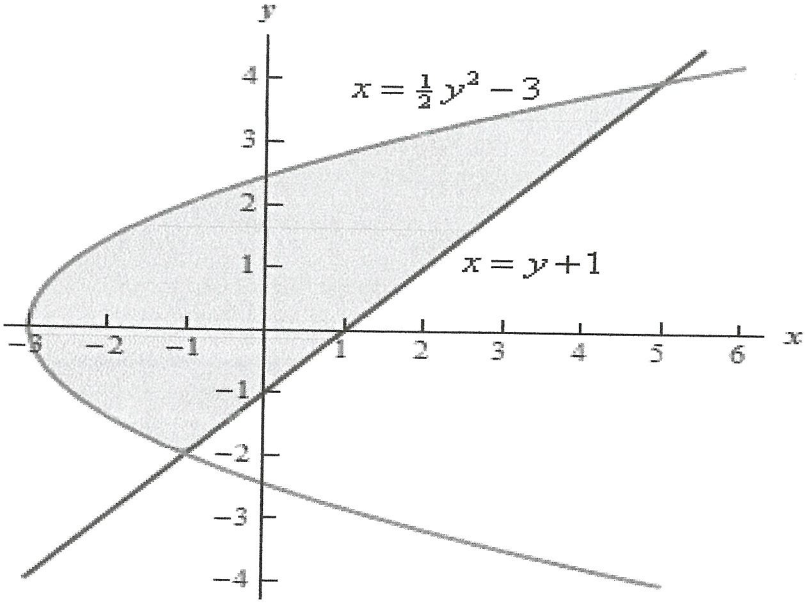

Figure Q2(a)

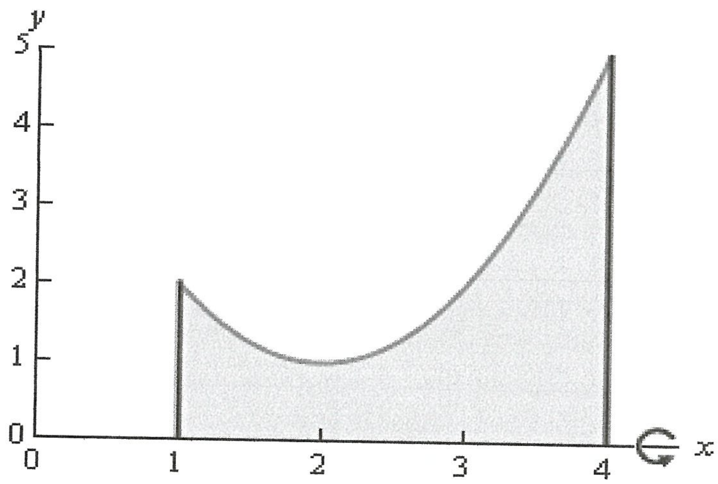

Figure Q2(b)

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Formula

Table 1 : Differentiation

$\frac{d}{dx}[c]=0$	$\frac{d}{dx}[\sin x]=\cos x$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos x]=-\sin x$
$\frac{d}{dx}[cu]=c\frac{du}{dx}$	$\frac{d}{dx}[\tan x]=\sec^2 x$
$\frac{d}{dx}[u\pm v]=\frac{du}{dx}\pm\frac{dv}{dx}$	$\frac{d}{dx}[\sec x]=\sec x\tan x$
$\frac{d}{dx}[uv]=u\frac{dv}{dx}+v\frac{du}{dx}$	$\frac{d}{dx}[\cot x]=-\csc^2 x$
$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}[\csc x]=-\csc x\cot x$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\frac{d}{dx}[\sin^{-1}x]=\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}[e^{nx}]=ne^{nx}$	$\frac{d}{dx}[\cos^{-1}x]=\frac{-1}{\sqrt{1-x^2}}$
Chain rule : $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$	$\frac{d}{dx}[\tan^{-1}x]=\frac{1}{1+x^2}$
Parametric differentiation : $\frac{dy}{dx}=\frac{dy}{dt}\Big/\frac{dx}{dt}=\frac{dy}{dt}.\frac{dt}{dx}$	$\frac{d}{dx}[\sec^{-1}x]=\frac{1}{\|x\|\sqrt{x^2-1}}$

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Table 2 : Integration

$\int c\, f(x)dx = c\, F(x) + C$	$\int \tan x\, dx = \ln\lvert \sec x \rvert + C$
$\int [f(x) \pm g(x)]dx = F(x) \pm G(x) + C$	$\int \sec^2 x\, dx = \tan x + C$
$\int x^r\, dx = \dfrac{x^{r+1}}{r+1} + C, (r \neq -1)$	$\int \csc^2 x\, dx = -\cot x + C$
$\int udv = uv - \int vdu$	$\int \csc x\, dx = -\ln\lvert \csc x + \cot x \rvert + C$
$\int \cos x\, dx = \sin x + C$	$\int \sec x \tan x\, dx = \sec x + C$
$\int \sin x\, dx = -\cos x + C$	$\int \csc x \cot x\, dx = -\csc x + C$
Integration by part : $\int udv = uv - \int vdu$	$\int \sec x\, dx = \ln\lvert \sec x + \tan x \rvert + C$

Area of region : $A = \int_a^b [f(x) - g(x)]dx$ or $A = \int_c^d [w(y) - v(y)]dy$

Volume (disc method) : $V = \pi \int_a^b [f(x)]^2\, dx$ or $V = \pi \int_c^d [f(y)]^2\, dy$

Arc length : $L = \int_a^b \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}\, dx$ or $L = \int_c^d \sqrt{1 + \left(\dfrac{dx}{dy}\right)^2}\, dy$

Simpson's rule : $\int_a^b f(x)\, dx \approx \dfrac{h}{3}\left[(f_0 + f_n) + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i\right]$; $n = \dfrac{b-a}{h}$

Trapezoidal rule : $\int_a^b f(x)\, dx \approx \dfrac{h}{2}\left[(f_0 + f_n) + 2\sum_{i=1}^{n-1} f_i\right]$; $n = \dfrac{b-a}{h}$

Partial Fraction : $\dfrac{a}{x^2 - 1} = \dfrac{A}{x+1} + \dfrac{B}{x-1}$