# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2023/2024

| | | |
|---|---|---|
| COURSE NAME | : | AIRCRAFT STABILITY AND CONTROL |
| COURSE CODE | : | BDU 21403 |
| PROGRAMME CODE | : | BDC |
| EXAMINATION DATE | : | JULY 2024 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | 1. PART A: ANSWER ALL QUESTIONS<br>2. PART B: ANSWER ONLY **ONE (1)** QUESTION FROM TWO (2) QUESTIONS<br>3. THIS FINAL EXAMINATION IS CONDUCTED VIA<br>☐ Open book<br>☒ Closed book<br>4. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK |

THIS QUESTION PAPER CONSISTS OF **TWELVE (12)** PAGES

**PART A**

Answer all questions in PART A.

**Q1** (a) State the performance specification for a second-order system and indicate the specification in a step response graph.

(5 marks)

(b) **Figure Q1.1** depicts a jet transport aircraft's stick-fixed longitudinal roots. Identify the roots of the corresponding longitudinal dynamic modes and determine the time for the amplitude to half, as well as the number of cycles.

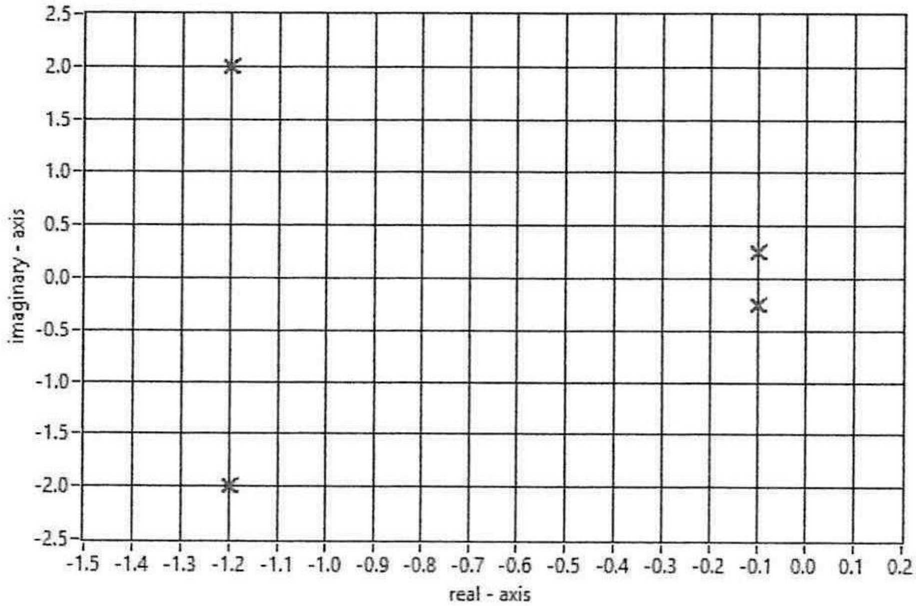



**Figure Q1.1 The stick-fixed longitudinal roots of a jet transport aircraft.**

(6 marks)

(c) An attitude control system for a satellite vehicle within the earth's atmosphere is shown in **Figure Q1.2**. The transfer functions of the system are given as follows:

$$G(s) = \frac{K(s+0.2)}{(s+0.9)(s-0.6)(s-0.1)}$$

$$G_c(s) = \frac{(s^2+4s+6.25)}{(s+4)}$$

Determine the range of gain, $K$, that results in a system with a settling time of less than 10 s and a damping ratio for the complex roots greater than 0.643. Provide a root locus sketch for the closed-loop system as $K$ varies from 0 to $\infty$ with the following calculations such as the asymptote angles, centroid and angle of departure/arrival to support your answer.

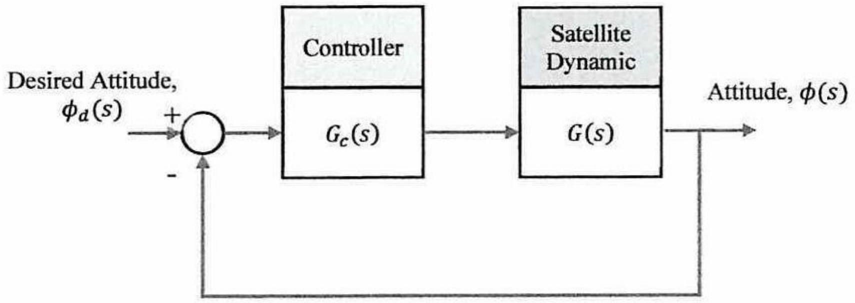



**Figure Q1.2 The block diagram for the satellite control system.**

(14 marks)

**Q2** In terms of flying and handling quality, the short-period response characteristics of an aircraft are particularly important. The governing equation of this simple motion is obtained from Newton's second law and is given as:

$$\Delta\ddot{\alpha} + 3\Delta\dot{\alpha} + 2\Delta\alpha = \Delta\delta_e$$

where $\Delta\alpha$ is the change in the angle of attack (assumption: the change in the angle of attack and pitch angles are identical), $\Delta\delta e$ is the change in elevator angle. Note that the initial condition of this simple motion is given as $\alpha(0) = 1$ and $\Delta\dot{\alpha} = 0$.

(a) Write the second-order differential equation in state-space form.

(6 marks)

(b) Find the solution to the state space model in **Q2(a)** using Paynter's numerical method. Use the time interval $\Delta t = 0.01$ to solve the numerical problem.

(8 marks)

(c) Plot the output response of the system, $\alpha(t)$ based on the solution obtained in **Q2(b)**. Indicate in the graph when the steady state value of the output response, $\alpha(t)$ is achieved.

(6 marks)

(d) Comment on the time response characteristics of the short-period motion obtained for this aircraft. Do your findings agree with the handling quality criteria shown in **Figure Q2.1**?

(5 marks)

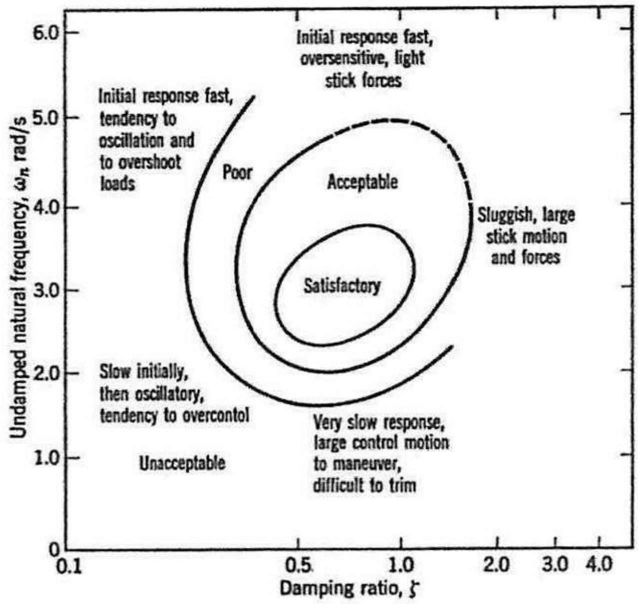



**Figure Q2.1 The short period flying quality.**

**Q3** (a) Describe the physical characteristics of the Dutch Roll stability mode.

(2 marks)

(b) The Dutch Roll motion can be approximated using the following equation:

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \dfrac{Y_\beta}{u_0} & -\left(1 - \dfrac{Y_r}{u_0}\right) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} \dfrac{Y_{\delta r}}{u_0} \\ N_{\delta r} \end{bmatrix} \delta r$$

Assume the aircraft has the following stability derivatives as follows:

$Y_\beta = -19.5$ ft/s$^2$ $\qquad Y_r = 1.3$ ft/s

$N_\beta = 1.5$ s$^{-2}$ $\qquad N_r = -0.21$ s$^{-1}$

$Y_{\delta r} = -4.7$ ft/s$^2$ $\qquad N_{\delta r} = 0.082$ s$^{-2}$

$u_0 = 400$ ft/s

(i) Determine the characteristic equation of the Dutch Roll mode.

(3 marks)

(ii) Determine the eigenvalues of the Dutch Roll mode.

(2 marks)

(iii) Determine the damping ratio, natural frequency, period, time to half amplitude, and the number of cycles to half amplitude for the Dutch Roll mode.

(5 marks)

(c) Determine the yaw rate to rudder input transfer function from the state space equation in **Q3(b)**.

(6 marks)

(d) Determine the yaw damper gain for the aircraft in **Q3(b)** that provides adequate damping in Dutch Roll mode. Use the block diagram in **Figure Q3.1** as the controller structure for the design problem. The system performance is expected to have a damping ratio, $\xi = 0.3$ and natural frequency, $\omega_n = 1$ rad/s. Consider the sensor used in the control system design to be a perfect device.

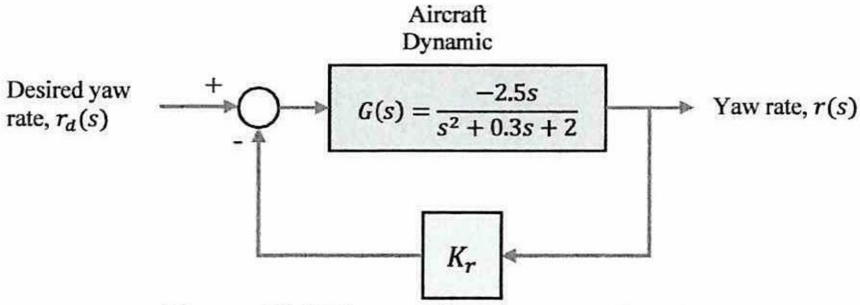



**Figure Q3.1 The yaw damper control system.**

(7 marks)

**PART B**

Answer only **ONE** question from Part B.

**Q4** Consider a pitch autopilot system with a stability augmentation mechanism for a fixed-wing based unmanned aerial system shown in **Figure Q4.1.**

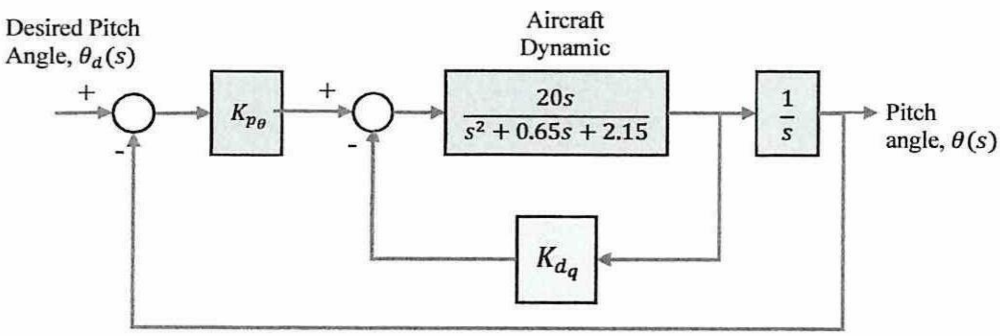



**Figure Q4.1 Pitch angle control system.**

(a) Determine the time response performance of the open-loop system and comment on any problems with the system.

(5 marks)

(b) Determine the proper controller gains for the aircraft to improve the system response characteristics so that the control system has the following performance: overshoot, $OS = 5\%$ and settling time, $T_s = 2$ sec. Evaluate the steady-state error performance of the compensated system and comment on any design problems.

(20 marks)

**Q5** A roll control system for a fixed wing aircraft is shown in **Figure Q5.1**. The transfer functions of the system are given as follows:

$$G(s) = \frac{1}{s(s+1)}$$

$$G_c(s) = \frac{K}{(s+9)}$$

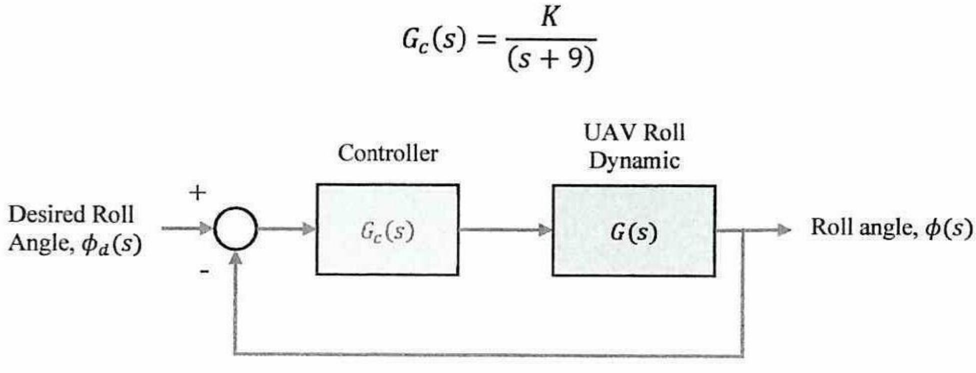



**Figure Q5.1 Simplified block diagram for roll angle control system.**

(a) **Figure Q5.2** shows the resulting root locus plot for the roll control system when operated at $\xi = 0.707$ which results in controller gain, $K = 4.04$ with the following complex poles, $s_{1,2} = -0.472 \pm 0.472j$. Determine the centroid, asymptote angles and imaginary axis intersection points of the root locus plot.

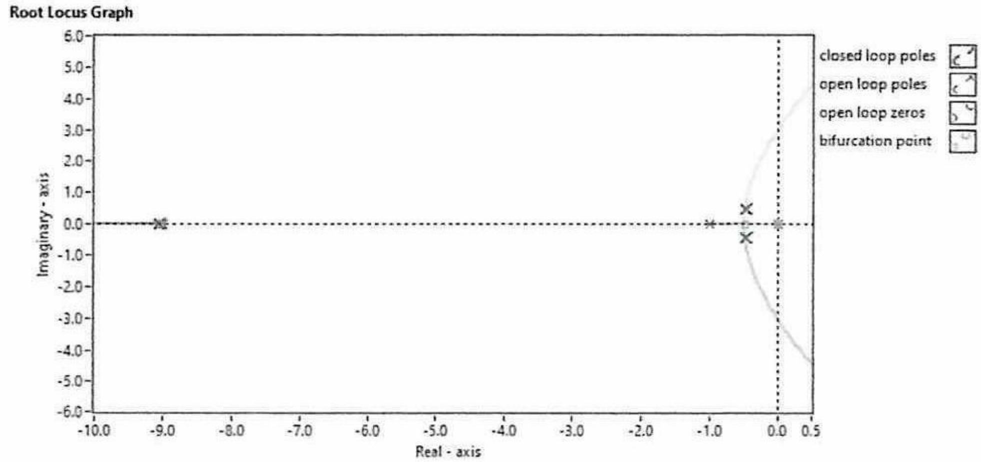



**Figure Q5.2 The root locus plot for the roll control system. The red cross mark symbols indicate the closed-loop poles of the system.**

(5 marks)

(b) Evaluate the time response characteristics of the uncompensated system in **Q5(a)**. If the PD controller design is intended to be used as a compensator, recommend the value of PD controller gains for the roll angle control system so that the system operates at a damping ratio of $\xi = 0.707$ with a double reduction in settling time. Provide a detailed root locus plot for the closed-loop system as $K$ varies from 0 to $\infty$ with necessary calculations such as the asymptote angles and centroid to support your answer.

(20 marks)

**-END OF QUESTION-**

**APPENDIX A**

**Key Equations**

The relevant equations used in this examination are given as follows:

1. The determinant of a 3 × 3 matrix:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \quad (1)$$

2. Partial fraction for $F(s)$ with real and distinct roots in the denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2}{(s+p_2)} + \cdots + \frac{K_m}{(s+p_m)} \quad (2)$$

3. Partial fraction for $F(s)$ with complex or imaginary roots in the denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2 s + K_3}{(s^2 + as + b)} + \cdots \quad (3)$$

4. General first-order transfer function:

$$G(s) = \frac{K}{s+a} \quad (4)$$

5. General second-order transfer function:

$$G(s) = \frac{K}{s^2 + 2\xi\omega_n s + \omega_n} \quad (5)$$

6. The closed-loop transfer function:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \quad (6)$$

where $G(s)$ is the transfer function of the open-loop system/forward path elements, and $H(s)$ is the transfer function in the feedback loop.

7. The final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \quad (7)$$

8. Time response:

$$T_r = \frac{2.2}{a} \quad (8)$$

$$T_s = \frac{4}{a} \quad (9)$$

$$\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\% \quad (10)$$

$$\xi = \frac{-\ln\left(\%\frac{OS}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\%\frac{OS}{100}\right)\right)^2}} \quad (11)$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega_d} \qquad (12)$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{\sigma} \qquad (13)$$

$$T_r = \frac{1.76\xi^3 - 0.417\xi^2 + 1.039\xi + 1}{\omega_n} \qquad (14)$$

$$P = \frac{2\pi}{\omega_d} \qquad (15)$$

$$t_{1/2} = \frac{0.693}{|\sigma|} \qquad (16)$$

$$N_{1/2} = 0.110\frac{|\omega_d|}{|\sigma|} \qquad (17)$$

9. Estimation of parameter $q$ (Paynter Numerical Method)

$$q = \max\left|A_{ij}\Delta t\right| \qquad (18)$$

10. Estimation of the integer value of $p$ (Paynter Numerical Method)

$$\frac{1}{p!}(nq)^p e^{nq} \leq 0.001 \qquad (19)$$

11. Numerical solution of state equation:

$$\mathbf{x}_{k+1} = \boldsymbol{M}\mathbf{x}_k + \boldsymbol{N}\boldsymbol{\eta}_k$$

with matrix $\boldsymbol{M}$ and $\boldsymbol{N}$ are given by the following matrix expansion: (20)

$$\boldsymbol{M} = e^{A\Delta t} = \boldsymbol{I} + A\Delta t + \frac{1}{2!}\boldsymbol{A}^2\Delta t^2 \ldots$$

$$\boldsymbol{N} = \Delta t\left(\boldsymbol{I} + \frac{1}{2!}\boldsymbol{A}\Delta t + \frac{1}{3!}\boldsymbol{A}^2\Delta t^2 + \cdots\right)\boldsymbol{B}$$

12. Characteristic equation of the closed loop system:

$$1 + KG(s)H(s) = 0 \qquad (21)$$

13. Asymptotes: angle and real-axis intercept:

$$\sigma = \frac{[\sum Real\ parts\ of\ the\ poles - \sum Real\ parts\ of\ the\ zeros]}{n-m} \qquad (22)$$

$$\phi_a = \frac{180°[2q+1]}{n-m} \qquad (23)$$

14. The solution to determine real axis break-in and breakaway points:

$$\frac{dK(\sigma)}{d\sigma} = 0 \qquad (24)$$

15. Final value of continuous linear time invariant (LTI) systems:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{Ax}(\boldsymbol{t}) + \boldsymbol{Bu}(\boldsymbol{t})$$
$$\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{Cx}(\boldsymbol{t}) \qquad (25)$$

In response to a step input $\boldsymbol{u}(\boldsymbol{t})$ with amplitude $R$ is:

$$\lim_{t\to\infty} \boldsymbol{y}(t) = -\boldsymbol{CA}^{-1}\boldsymbol{BR}$$

16. An alternative solution to find real axis break-in and breakaway points:

$$\sum \frac{1}{\sigma + z_i} = \sum \frac{1}{\sigma + p_i} \quad (26)$$

17. The angle of departure of the root locus from a pole of $G(s)H(s)$:

$$\theta = 180° + \sum (angles\ to\ zeros) - \sum (angles\ to\ poles) \quad (27)$$

18. The angle of arrival at a zero:

$$\theta = 180° - \sum (angles\ to\ zeros) + \sum (angles\ to\ poles) \quad (28)$$

19. The steady-state error:

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) \quad (29)$$

where the error signal is given as:

$$E(s) = \frac{1}{1 + K(s)G(s)H(s)} \times U(s) \quad (30)$$

Denominator of equation, $1 + K(s)G(s)H(s) = 0$ indicates the characteristic equation of the closed loop system.

20. The characteristic equation for the standard form of the second-order differential equation:

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0$$

The roots of the characteristic equation are: (31)

$$\lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{1 - \xi^2} \cdot i$$

$$\lambda_{1,2} = \sigma \pm \omega_d$$

21. The calculation of controller gains using the Ziegler-Nichols method:

**Table 1** The Ziegler-Nichols tuning method. (32)

| **Control Type** | $K_p$ | $K_I$ | $K_D$ |
|---|---|---|---|
| P | $0.5K_u$ | - | - |
| PI | $0.45K_u$ | $1.2\,K_p/T_u$ | - |
| PD | $0.8K_u$ | - | $K_PT_u/8$ |
| Classic PID | $0.6K_u$ | $2\,K_p/T_u$ | $K_PT_u/8$ |
| Pessen Integral Rule | $0.7K_u$ | $2.5\,K_p/T_u$ | $3K_PT_u/20$ |
| Some Overshoot | $0.33K_u$ | $2\,K_p/T_u$ | $K_PT_u/3$ |
| No Overshoot | $0.2K_u$ | $2\,K_p/T_u$ | $K_PT_u/3$ |

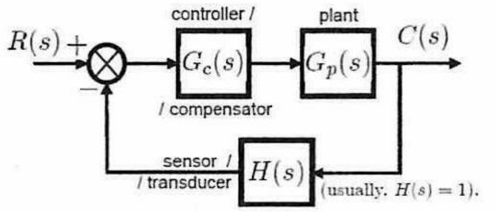
22. Conversion from the state-space model to transfer function model: (33)

$$G(s) = \boldsymbol{C}\frac{adj(\boldsymbol{sI} - \boldsymbol{A})}{\det(\boldsymbol{sI} - \boldsymbol{A})}\boldsymbol{B}$$

23. Final value theorem for transfer function model: (34)

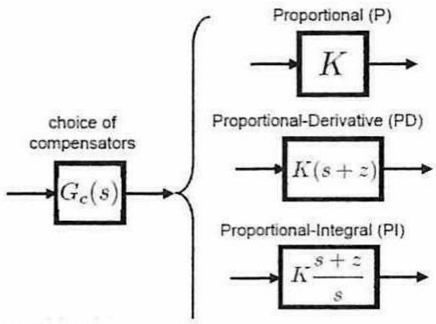
$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

## 24. Compensator design using the Root Locus:




generic system block diagram with controller/compensator and feedback

purpose: to improve the system's dynamics by proper choice of the controller TF and gain




Open–Loop TF: $KG_p(s)G_c(s)H(s)$ Closed–Loop TF: $\dfrac{KG_p(s)G_c(s)}{1+KG_p(s)G_c(s)H(s)}$

## 25. Root Locus sketching rules:

1. The root locus contours are symmetrical about the real axis.
2. The number of separate branches of the root locus plot is equal to the number of poles of the transfer function $G(s)H(s)$. Branches of the root locus originate at the poles of $G(s)H(s)$ for $k = 0$ and terminate at either the open-loop zeroes or at infinity for $k = \infty$. The number of branches that terminate at infinity is equal to the difference between the number of poles and zeroes of the transfer function $G(s)H(s)$, where $n$ = number of poles and $m$ = number of zeros.
3. Segments of the real axis that are part of the root locus can be found in the following manner: Points on the real axis that have an odd number of poles and zeroes to their right are part of the real axis portion of the root locus.
4. The root locus branches that approach the open-loop zeroes at infinity do so along straight-line asymptotes that intersect the real axis at the center of gravity of the finite poles and zeroes. Mathematically this can be expressed as

$$\sigma = \left[\sum \text{Real parts of the poles} - \sum \text{Real parts of the zeroes}\right] \Big/ (n-m)$$

   where $n$ is the number of poles and $m$ is the number of finite zeroes.
5. The angle that the asymptotes make with the real axis is given by

$$\phi_a = \frac{180°[2q+1]}{n-m}$$

   for $q = 0, 1, 2, \ldots, (n-m-1)$
6. The angle of departure of the root locus from a pole of $G(s)H(s)$ can be found by the following expression:

$$\phi_p = \pm 180°(2q+1) + \phi \qquad q = 0, 1, 2, \ldots$$

   where $\phi$ is the net angle contribution at the pole of interest due to all other poles and zeroes of $G(s)H(s)$. The arrival angle at a zero is given by a similar expression:

$$\phi_z = \pm 180°(2q+1) + \phi \qquad q = 0, 1, 2, \ldots$$

   The angle $\phi$ is determined by drawing straight lines from all the poles and zeroes to the pole or zero of interest and then summing the angles made by these lines.
7. If a portion of the real axis is part of the root locus and a branch is between two poles, the branch must break away from the real axis so that the locus ends on a zero as $k$ approaches infinity. The breakaway points on the real axis are determined by solving

$$1 + GH = 0$$

   for $k$ and then finding the roots of the equation $dk/ds = 0$. Only roots that lie on a branch of the locus are of interest.

26. The Laplace transform theorems:

| Laplace transforms – Table | | | |
|---|---|---|---|
| $f(t) = L^{-1}\{F(s)\}$ | $F(s)$ | $f(t) = L^{-1}\{F(s)\}$ | $F(s)$ |
| $a \quad t \geq 0$ | $\frac{a}{s} \quad s > 0$ | $\sin \omega t$ | $\frac{\omega}{s^2 + \omega^2}$ |
| $at \quad t \geq 0$ | $\frac{a}{s^2}$ | $\cos \omega t$ | $\frac{s}{s^2 + \omega^2}$ |
| $e^{-at}$ | $\frac{1}{s + a}$ | $\sin(\omega t + \theta)$ | $\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$ |
| $te^{-at}$ | $\frac{1}{(s + a)^2}$ | $\cos(\omega t + \theta)$ | $\frac{s \cos \theta - \omega \sin \theta}{s^2 + \omega^2}$ |
| $\frac{1}{2}t^2 e^{-at}$ | $\frac{1}{(s + a)^3}$ | $t \sin \omega t$ | $\frac{2\omega s}{(s^2 + \omega^2)^2}$ |
| $\frac{1}{(n-1)!} t^{n-1} e^{-at}$ | $\frac{1}{(s + a)^n}$ | $t \cos \omega t$ | $\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$ |
| $e^{at}$ | $\frac{1}{s - a} \quad s > a$ | $\sinh \omega t$ | $\frac{\omega}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$ |
| $te^{at}$ | $\frac{1}{(s - a)^2}$ | $\cosh \omega t$ | $\frac{s}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$ |
| $\frac{1}{b - a}(e^{-at} - e^{-bt})$ | $\frac{1}{(s + a)(s + b)}$ | $e^{-at} \sin \omega t$ | $\frac{\omega}{(s + a)^2 + \omega^2}$ |
| $\frac{1}{a^2}[1 - e^{-at}(1 + at)]$ | $\frac{1}{s(s + a)^2}$ | $e^{-at} \cos \omega t$ | $\frac{s + a}{(s + a)^2 + \omega^2}$ |
| $t^n$ | $\frac{n!}{s^{n+1}} \quad n = 1,2,3..$ | $e^{at} \sin \omega t$ | $\frac{\omega}{(s - a)^2 + \omega^2}$ |
| $t^n e^{at}$ | $\frac{n!}{(s - a)^{n+1}} \quad s > a$ | $e^{at} \cos \omega t$ | $\frac{s - a}{(s - a)^2 + \omega^2}$ |
| $t^n e^{-at}$ | $\frac{n!}{(s + a)^{n+1}} \quad s > a$ | $1 - e^{-at}$ | $\frac{a}{s(s + a)}$ |
| $\sqrt{t}$ | $\frac{\sqrt{\pi}}{2s^{3/2}}$ | $\frac{1}{a^2}(at - 1 + e^{-at})$ | $\frac{1}{s^2(s + a)}$ |
| $\frac{1}{\sqrt{t}}$ | $\sqrt{\frac{\pi}{s}} \quad s > 0$ | $f(t - t_1)$ | $e^{-t_1 s} F(s)$ |
| $g(t) \cdot p(t)$ | $G(s) \cdot P(s)$ | $f_1(t) \pm f_2(t)$ | $F_1(s) \pm F_2(s)$ |
| $\int f(t)dt$ | $\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$ | $\delta(t)$ *unit impulse* | $1$ *all s* |
| $\frac{df}{dt}$ | $sF(s) - f(0)$ | $\frac{d^2 f}{df^2}$ | $s^2 F(s) - sf(0) - f'(0)$ |
| $\frac{d^n f}{dt^n}$ | $s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \cdots - f^{n-1}(0)$ | | |