UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAM
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	CALCULUS FOR ENGINEER
COURSE CODE	:	BDA 14403
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. PART A: ANSWER ALL QUESTIONS PART B: ANSWER **TWO (2)** FROM THREE (3) QUESTIONS ONLY 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

PART A

Q1 (a) State the domain and range of each of the following functions

i. $z = x^2\sqrt{y+4}$

(3 marks)

ii. $z = \dfrac{1}{xy+2}$

(3 marks)

iii. $z = e^{\sqrt{x+y}}$

(3 marks)

(b) Determine the following limit if it exists. If not, show that the limit does not exist.

i. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^4}{x^2+y^8}$

(5 marks)

ii. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4-y^4}{x^2+y^2}$

(6 marks)

Q2 (a) If $z = \dfrac{1}{2}\ln(x^2+y^2)$, identify that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 1$

(6 marks)

(b) Calculate $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for function $z^2 + z\sin(xy^2) = 0$ if $z = f(x,y)$ is implicitly defined as a function of x and y.

(6 marks)

(c) Syrup is being withdrawn from its container at a rate of 104 cm^3/min. Simultaneously, it is being poured into a cone at a rate of increasing radius 2 cm/min. When 30 cm^3 of syrup has been withdrawn, the radius of the cone measures 6 cm. Determine the rate at which the height of the syrup is increasing at that moment.

(8 marks)

Q3 Let $f(x,y) = x^2y$.

(a) Calculate $\nabla f(3,2)$.

(8 marks)

(b) Identify the derivative of f in the direction of $(1,2)$ at the point $(3,2)$.

(8 marks)

(c) Find the directional derivatives of f at the point (3,2) in the direction of (2,1).

(4 marks)

PART B

Q4 (a) Identify the line integral $\int_C (x+y)dx + xydy$ where C consists of two line segments, from (0,0) to (1,0) along the line $y = 0$ and from (1,0) to (1,1) along the line $x = 1$

(5 marks)

(b) Solve the integral $\int_c (x^2 - y^2)dx - 2xy\, dy$ along the parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$.

(7 marks)

(c) Use Green's Theorem to calculate $\int_C (2xy - y^2)dx + (x^2 - y^2)dy$, where C is the boundary of the region enclosed by $y = x$ and $y = x^2$. Assume that the curve c is traversed in a counterclockwise manner.

(8 marks)

Q5 (a) By using a double integral, Identify the volume of solid enclosed by a cylinder $y^2 + x^2 = 1$, plane $z = y$ and yz-plane in the first octant.

(8 marks)

(b) Calculate the volume of the solid tetrahedron enclosed by $2x + y + z = 4$ and the coordinate planes by using a triple integral.

(12 marks)

Q6 (a) Find the derivative of the given function by using the appropriate formula

(i) $f(x) = (x^{100} + 2x^{50} - 3)(7x^8 + 20x + 5)$.

(2 marks)

(ii) $g(x) = \dfrac{x^5 - x + 2}{x^3 + 7}$

(3 marks)

(b) Oil from an uncapped well is radiating outward in the form of a circular film on the surface of the water. If the radius of the circle is increasing at the rate of 1.5 meters per minute, Please identify the velocity of the area of the oil film growing at the instant when the radius is 150 meters.

(7 marks)

(c) Solve the following by using polar coordinates

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \frac{r\cos\theta}{\sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}}\, dy\, dx.$$

(8 marks)

-END OF QUESTIONS –

APPENDIX A

FORMULA

Total Differential

For function $z = f(x, y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change

For function $z = f(x, y)$, the relative change in z is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x, y)$ by an equation of the form $F(x, y, z) = 0$, where $F(x, y, f(x, y)) = 0$ for all (x, y) in the domain of f, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
   $f(x, y)$ has a local maximum value at (a,b)

b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
   $f(x, y)$ has a local minimum value at (a,b)

c. If $D < 0$
   $f(x, y)$ has a saddle point at (a,b)

d. If $D = 0$
   The test is inconclusive

Surface Area

$$\text{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$$

APPENDIX B

FORMULA

Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)rdzdrd\theta$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi d\rho d\phi d\theta$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint_R \delta(x,y)dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint_R y\delta(x,y)dA$

b. About y-axis, $M_y = \iint_R x\delta(x,y)dA$

APPENDIX C

FORMULA

Centre of Mass
Non-Homogeneous Lamina:
$$(\bar{x},\bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

Centroid
Homogeneous Lamina:
$$\bar{x} = \frac{1}{Area\,of\quad R}\iint_R x\,dA \text{ and } \bar{y} = \frac{1}{Area\,of\quad R}\iint_R y\,dA$$

Moment Inertia:

a. $I_y = \iint_R x^2\delta(x,y)dA$

b. $I_x = \iint_R y^2\delta(x,y)dA$

c. $I_o = \iint_R (x^2+y^2)\delta(x,y)dA$

In 3-D: Solid
Given that $\delta(x,y,z)$ is a density of solid

Mass, $m = \iiint_G \delta(x,y,z)dV$

If $\delta(x,y,z)=c$, where c is a constant, $m = \iiint_G dA$ is volume.

Moment of Mass

a. About yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)dV$

b. About xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)dV$

c. About xy-plane, $M_{xy} = \iiint_G z\delta(x,y,z)dV$

Centre of Gravity
$$(\bar{x},\bar{y},\bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$$

APPENDIX D

FORMULA

Moment Inertia

a. About x-axis, $I_x = \iiint_G (y^2 + z^2)\delta(x, y, z)\,dV$

b. About y-axis, $I_y = \iiint_G (x^2 + z^2)\delta(x, y, z)\,dV$

c. About z-axis, $I_z = \iiint_G (x^2 + y^2)\delta(x, y, z)\,dV$

Directional Derivative

$D_u f(x, y) = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot \mathbf{u}$

Del Operator

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Gradient of $\phi = \nabla\phi$

Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let C is smooth curve defined by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

APPENDIX E

FORMULA

The **Binormal Vector**, $\mathbf{B}(t)=\mathbf{T}(t)\times\mathbf{N}(t)$

Curvature

$$\kappa=\frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho=\frac{1}{\kappa}$$

Green's Theorem

$$\oint_C Mdx+Ndy=\iint_R\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dA$$

Gauss's Theorem

$$\iint_S \mathbf{F}\cdot\mathbf{n}dS=\iiint_G \nabla\cdot\mathbf{F}dV$$

Stoke's Theorem

$$\oint_C \mathbf{F}\cdot dr=\iint_S(\nabla\times\mathbf{F})\cdot\mathbf{n}dS$$

Arc Length

If $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}, t\in[a,b]$, hence, the **arc length**,

$$s=\int_a^b\|\mathbf{r}'(t)\|\,dt=\int_a^b\sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\,dt$$