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Universiti Tun Hussein Onn Malaysia

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2016/2017

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|---|---|---|
| COURSE NAME | : | ALGEBRA |
| COURSE CODE | : | DAS 10103 |
| PROGRAMMECODE | : | DAT |
| EXAMINATION DATE | : | JUNE 2017 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWER FIVE (5) QUESTIONS ONLY. |

TERBUKA

THIS QUESTION PAPER CONSISTS OF SEVEN (7) PAGES

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**Q1** (a) (i) Simplify $\left(\dfrac{10abc}{(a^2b)(2bc^3)}\right)^2$.

(3 marks)

(ii) Given $p^{3x} = q^{x+2y}$ and $p^{xy} = q^{x-y}$. Prove that $(x+2y)(xy)=(x-y)(3x)$.

(6 marks)

(b) (i) Solve the equation $\log_x 135 = \log_x 5 + 3$

(5 marks)

(ii) Solve $3^{\log_2 x} = 243$

(3 marks)

(c) Simplify the expression below. Assume that *x*, *y* and *z* are positive.

$$\sqrt{25x^3y} \cdot \sqrt{10x^2y^3z^5}$$

(3 marks)

**Q2** (a) Find the root of the equation $f(x) = 2x^2 - x - 2 = 0$ in the interval $[1, 2]$ using Secant method. Iterate until $|f(x_i)| < \varepsilon = 0.005$. Show your calculation in three decimal places.

(9 marks)

(b) Express $\dfrac{4x-2}{(x+3)(x-2)^2}$ in the form of partial fraction.

(6 marks)

(c) Using Binomial expansion, find the first three terms of $(2x+y)^5$

(5 marks)

**Q3** (a) (i) Find the pattern of the following sequence 1, 6, 11, . . .

(3 marks)

(ii) Given the sum of the first *n* terms of an arithmetic sequence 2, 5, 8… is 100. Find the value of *n*.

(3 marks)

(b) Given that the $n^{\text{th}}$ term of arithmetic sequence is $T_n = 23 + 2(n-1)$.

(i) Find the value of first term, $a$ and its common difference, $d$.
(3 marks)

(ii) Find $S_{10}$.
(3 marks)

(c) A geometric sequence is defined as $30, 20, \frac{40}{3}, \frac{80}{9}, \ldots$

(i) Find the value of common ratio, $r$.
(2 marks)

(ii) Calculate the tenth term, $a_{10}$.
(2 marks)

(iii) State whether this series converges or diverges. Give your reason.
(2 marks)

(iv) If it is converges, evaluate its summation, $S_\infty$.
(2 marks)

**Q4** (a) By using sum and difference identities, simplify and evaluate
$\sin(45^\circ + 30^\circ) + \sin(45^\circ - 30^\circ)$
(5 marks)

(b) Without using calculator, find the value of:

(i) $\cos 120°$ by using the double angle formula.
(4 marks)

(ii) $\sin 15°$ by using the half angle formula.
(4 marks)

(c) Given $3\cos\theta + 4\sin\theta = r\sin(\theta + \alpha)$ and $0 \le \theta \le 2\pi$.

(i) Find $r$ and $\alpha$.
(2 marks)

(ii) Thus, find the value of $\theta$ if $3\cos\theta + 4\sin\theta = 1$.
(5 marks)

**Q5** (a) Given $\boldsymbol{A}=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix}5 & 20\\ -7 & 11\end{pmatrix}$ and $\boldsymbol{C}=\begin{pmatrix}10 & -6\\ 4 & -8\end{pmatrix}$.

(i) Calculate $\boldsymbol{AB}+\boldsymbol{C}$.

(4 marks)

(ii) Show that $(\boldsymbol{B}+\boldsymbol{C})^{T}=\boldsymbol{B}^{T}+\boldsymbol{C}^{T}$.

(3 marks)

(b) Given

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 6\\ 2x & - & y & - & 2z & = & 6\\ 3x & + & 2y & - & z & = & 8 \end{array}$$

(i) Write the matrix equation $\boldsymbol{AX}=\boldsymbol{B}$ of the system equation.

(3 marks)

(ii) Solve the above system for $x$, $y$, and $z$ by using the Gauss-Jordan elimination

method, start with the following operation:

$$\left(\begin{array}{ccc|c}1 & 1 & 1 & 6\\ 2 & -1 & -2 & 6\\ 3 & 2 & -1 & 8\end{array}\right)\xrightarrow{R2-2R1}\left(\begin{array}{ccc|c}1 & 1 & 1 & 6\\ \_ & \_ & \_ & \_\\ 3 & 2 & -1 & 8\end{array}\right)\xrightarrow{R3-3R1}\left(\begin{array}{ccc|c}1 & 1 & 1 & 6\\ \_ & \_ & \_ & \_\\ \_ & \_ & \_ & \_\end{array}\right)$$

$$\xrightarrow{R1+R3}\left(\begin{array}{ccc|c}1 & 0 & -3 & -4\\ \_ & \_ & \_ & \_\\ \_ & \_ & \_ & \_\end{array}\right)\xrightarrow{-R3\leftrightarrow R2}\left(\begin{array}{ccc|c}1 & 0 & -3 & -4\\ \_ & \_ & \_ & \_\\ \_ & \_ & \_ & \_\end{array}\right)\xrightarrow{?}\ldots$$

(10 marks)

**Q6** (a) Given that $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, such that A is an acute angle and B is an obtuse angle. Without using calculator, find

(i) $\cos A$

(2 marks)

(ii) $\tan B$

(3 marks)

(iii) $\tan A + \cos B$

(2 marks)

(iv) $\sin (A + B)$

(4 marks)

(v) $\tan (A - B)$

(5 marks)

(b) Given $\begin{pmatrix} 1 & 0 & 3 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ -5 \end{pmatrix}$.

By using Elementary Row Operation (ERO) or other method, find $x$, $y$ and $z$.

(4 marks)

**-END OF QUESTIONS –**

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**FORMULA**

**Polynomials**

$$\log_a x = \frac{\log_a x}{\log_a b}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c, \qquad x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

**Sequence and Series**

$$\sum_{k=1}^{n} c = cn, \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$u_n = a + (n-1)d \quad S_n = \frac{n}{2}[2a + (n-1)d], \qquad S_n = \frac{n}{2}(a + u_n)$$

$$u_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r-1}, r > 1 \textbf{ OR } S_n = \frac{a(1-r^n)}{1-r}, r < 1, \quad S_\infty = \frac{a}{1-r}.$$

$$u_n = S_n - S_{n-1}$$

$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots\ldots$$

**Trigonometry**

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \quad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \quad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

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## FORMULA

$$\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}\,,\quad \cos\frac{\theta}{2}=\pm\sqrt{\frac{1+\cos\theta}{2}}\,,\quad \tan\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$a\sin\theta+b\cos\theta=r\sin(\theta+\alpha)=r(\sin\theta\cos\alpha+\cos\theta\sin\alpha)=(r\cos\alpha)\sin\theta+(r\sin\alpha)\cos\theta$$

and

$a=r\cos\alpha$ and $b=r\sin\alpha$

$$x_1^{(k+1)}=\frac{b_1-a_{12}x_2^{(k)}-a_{13}x_3^{(k)}}{a_{11}}\,,\quad x_2^{(k+1)}=\frac{b_2-a_{21}x_1^{(k+1)}-a_{23}x_3^{(k)}}{a_{22}}\,,\quad x_3^{(k+1)}=\frac{b_3-a_{31}x_1^{(k+1)}-a_{32}x_2^{(k+1)}}{a_{33}}$$

**Matrices**

$$A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix},\ |A|=a_{11}\begin{vmatrix}a_{22}&a_{23}\\a_{32}&a_{33}\end{vmatrix}-a_{12}\begin{vmatrix}a_{21}&a_{23}\\a_{31}&a_{33}\end{vmatrix}+a_{13}\begin{vmatrix}a_{21}&a_{22}\\a_{31}&a_{32}\end{vmatrix}$$