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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	DIGITAL SIGNAL PROCESSING
COURSE CODE	:	BEJ 30603/BEV 30603
PROGRAMME CODE	:	BEJ/BEV
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)**

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Q1 Discrete signal is a time series consisting of a sequence of quantities. It can be represented as a sequence of numbers, which can be naturally discrete in time or it can be obtained from periodic sampling of a continuous-time signal.

(a) Define an even symmetric and an odd symmetric discrete signal.

(4 marks)

(b) Determine the mathematical expression using ramp function for the signal given by:

$$x[n] = 6tri\left(\frac{n+1}{3}\right) - 2tri\left(\frac{n-4}{2}\right)$$

You must provide a validation calculation for that expression.

(10 marks)

(c) Calculate the autocorrelation for the signal below:

$$s[n] = 5(0.3^n)u[n]$$

(6 marks)

Q2 Sampling is the first technique in digital signal processing to convert an analog signal to a digital signal.

(a) With the aid of a diagram, explain an ideal sampled signal.

(5 marks)

(b) Referring to this equation:

$$x(t) = 2\cos(2\pi t + 1) + \cos(4\pi t - 0.5)$$

(i) Calculate and plot the first **SIX (6)** samples of the discrete signal, $x[n]$ with a sampling frequency of 10 Hz.

(7 marks)

(ii) With a parameter of 4volts dynamic range, quantization using rounding technique and 3-bit system, calculate the digital signal of $x_c[n]$.

(5 marks)

(iii) Calculate the quantization error, $E_q[n]$.

(2 marks)

(iv) Suggest **ONE (1)** way to minimize the quatization error in Digital Signal Processing (DSP).

(1 mark)

Q3 The Discrete Fourier Transform (DFT) is one of the most powerful tools in digital signal processing, and it enables us to find the spectrum of a finite-duration signal.

(a) A discrete signal is given by $r[n] = \{ \overset{\Downarrow}{1}, 3, 1, -1\}$ and its Discrete Fourier Transform (DFT) is $R_{DFT}[k] = \left\{ \overset{\Downarrow}{4}, -4j, 0,\ 4j \right\}$. Determine:

(i) $s[n] = r^{*}[n]$ and its DFT.

(2 marks)

(ii) $t[n] = r[n]r[n]$ and its DFT.

(4 marks)

(b) Assume $x[n] = \{ \overset{\Downarrow}{0}.3, -1, 4, 6\}$ as an input signal of a digital system. Calculate the DFT of this signal using Decimation in Frequency (DIF) Fast Fourier Transform (FFT) algorithm.

(14 marks)

Q4 In mathematics and signal processing, the Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation.

(a) Define the stability of a digital system and how it relates to the region of convergence (ROC) in Z-transform

(3 marks)

(b) Determine the Z-transform, ROC and stability of the following system:

(i) $x_1[n] = \begin{cases} \left(\frac{1}{3}\right)^n & n \geq 0 \\ \left(\frac{1}{2}\right)^n & n < 0 \end{cases}$

(4 marks)

(ii) $x_2[n] = (2n-3)(2)^{n+2}u[n]$

(3 marks)

(c) Input, $x[n]$ and output, $y[n]$ signals of a causal system, $h[n]$ are given as follows:

$$x[n] = \left[\left(\frac{1}{2}\right)^n u[n]\right] - \left[\frac{1}{4}\left(\frac{1}{2}\right)^{n-1} u[n-1]\right]$$

$$y[n] = \left(\frac{1}{3}\right)^n u[n]$$

Determine the system impulse response, $h[n]$. Then, identify the stability condition and support your answer with an appropriate justification.

(10 marks)

Q5 The infinite impulse response (IIR) filter design primarily concentrates on the filter's magnitude response and regards the phase response as secondary.

(a) State **THREE (3)** advantages of bilinear transformation.

(3 marks)

(b) Convert $H(s) = \dfrac{3s^2 + 12s}{s^2 + 6s + 8}$ to $H(z)$ by using an appropriate response-invariant transformation for the given input as shown in **Figure Q5.1**.

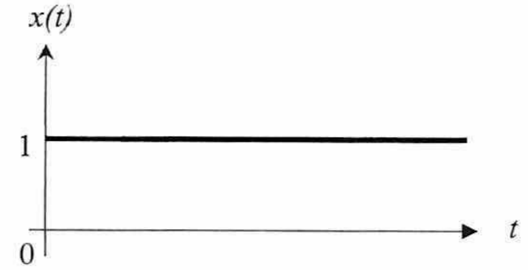

Figure Q5.1

(7 marks)

(c) The magnitude spectrum of the desired bandstop filter, which operates at a sampling frequency of 20 kHz, is shown in **Figure Q5.2**. Formulate the digital transfer function $H(z)$ if $H(s) = \frac{1}{s+1}$.

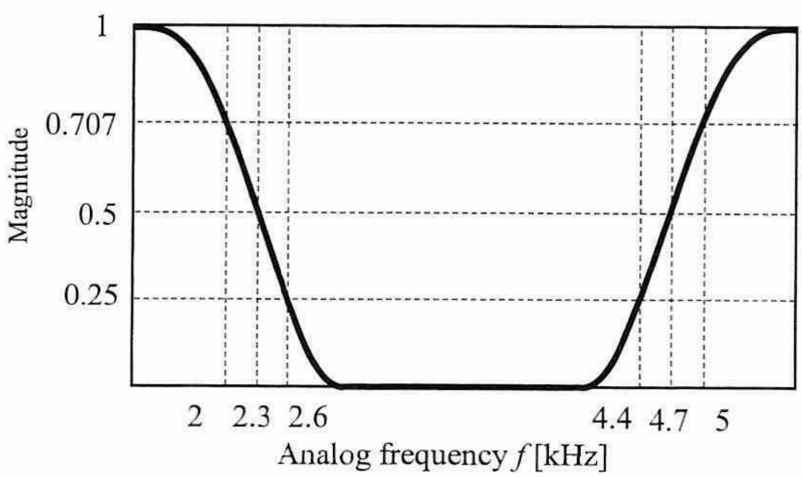

Figure Q5.2

(10 marks)

- END OF QUESTIONS -

APPENDIX A

Table 1: Properties of the *N*-Sample DFT

Property	**Signal**	**DFT**
Shift	$x[n-n_o]$	$X_{DFT}[k]e^{-j2\pi k n_o/N}$
Shift	$x[n-0.5N]$	$(-1)^k X_{DFT}[k]$
Modulation	$x[n]e^{j2\pi n k_o/N}$	$X_{DFT}[k-k_o]$
Modulation	$(-1)^n x[n]$	$X_{DFT}[k-0.5N]$
Folding	$x[-n]$	$X_{DFT}[-k]$
Product	$x[n]y[n]$	$\frac{1}{N}X_{DFT}[k]\otimes Y_{DFT}[k]$
Convolution	$x[n]\otimes y[n]$	$X_{DFT}[k]Y_{DFT}[k]$
Correlation	$x[n]\otimes\otimes y[n]$	$X_{DFT}[k]Y^*_{DFT}[k]$
Central Ordinates	$x[0]=\frac{1}{N}\sum_{k=0}^{N-1}X_{DFT}[k]$, $\quad X_{DFT}[0]=\sum_{n=0}^{N-1}x[n]$	
Central Ordinates	$x\left[\frac{N}{2}\right]=\frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (N even), $X_{DFT}\left[\frac{N}{2}\right]=\sum_{n=0}^{N-1}(-1)^n x[n]$ (N even)	

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Table 2: Properties of the z- transform

Property	**Signal**	**z-transform**
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(z) + a_2X_2(z)$
Time reversal	$x(-n)$	$X(z^{-1})$
Time shifting	i) $x(n-k)$ ii) $x(n+k)$	i) $z^{-k}X(z)$ ii) $z^{k}X(z)$
Convolution	$x_1(n) * x_2(n)$	$X_1(z)X_2(z)$
Correlation	$r_{x_1x_2}(l) = \sum_{n=-\infty}^{\infty} x_1(n)x_2(n-l)$	$R_{x_1x_2}(z) = X_1(z)X_2(z^{-1})$
Scaling	$a^n x(n)$	$X(a^{-1}z)$
Differentiation	$nx(n)$	$z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$
Time differentiation	$x(n) - x(n-1)$	$X(z)(1 - z^{-1})$
Time integration	$\sum_{k=0}^{\infty} X(k)$	$X(z) = \left(\frac{z}{z-1}\right)$
Initial value theorem	$\lim_{n \to 0} x(n)$	$\lim_{\lvert z\rvert \to \infty} X(z)$
Final value theorem	$\lim_{n \to \infty} x(n)$	$\lim_{\lvert z\rvert \to 1} \left(\frac{z-1}{z}\right) X(z)$

Table 3: z-Transform Pairs

Signal $x(t)$	Sequence $x(n)$	z-Transform $X(z)$
$\delta(t)$	$\delta(n)$	1
$\delta(t - k)$	$\delta(n - k)$	z^{-k}
$u(t)$	$u(n)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$
	$-u(-n - 1)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$
$r(t) = tu(t)$	$nu(n)$	$\frac{z^{-1}}{(1-z^{-1})^2} = \frac{z}{(z-1)^2}$
	$a^n u(n)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$
	$-a^n u(-n - 1)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$
	$na^n u(n)$	$\frac{az}{(z-a)^2}$
	$-na^n u(-n - 1)$	$\frac{az}{(z-a)^2}$
e^{-at}	e^{-an}	$\frac{1}{1-e^{-a}z^{-1}} = \frac{z}{z-e^{-a}}$
t^2	$n^2 u(n)$	$z^{-1}\frac{(1+z^{-1})}{(1-z^{-1})^3} = \frac{z(z+1)}{(z-1)^3}$
te^{-at}	ne^{-an}	$\frac{z^{-1}e^{-a}}{(1-e^{-a}z^{-1})^2} = \frac{ze^{-a}}{(z-e^{-a})^2}$
$sin\omega_o t$	$sin\omega_o n$	$\frac{z\sin\omega_o}{z^2 - 2z\cos\omega_o + 1}$
$cos\omega_o t$	$cos\omega_o n$	$\frac{z(z-\cos\omega_o)}{z^2 - 2z\cos\omega_o + 1}$

Table 4: Digital- to- digital Transformations

Form	Band Edges	Mapping $z \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-\alpha}{1-\alpha z}$	$\alpha = \frac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\frac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \frac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\frac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \frac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha K}{K+1}$, $A_2 = \frac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\frac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha}{K+1}$, $A_2 = \frac{1-K}{1+K}$

Table 5: Direct Analog- to- digital Transformations for Bilinear Design

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-1}{C(z+1)}$	$C = \tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\frac{C(z+1)}{z-1}$	$C = \tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1 < \Omega_0 < \Omega_2$	$\frac{z^2-2\beta z+1}{C(z^2-1)}$	$C = \tan[0.5(\Omega_2 - \Omega_1)]$, $\beta = \cos\Omega_0$ or $\beta = \frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$
Lowpass to bandstop	$\Omega_1 < \Omega_0 < \Omega_2$	$\frac{C(z^2-1)}{z^2-2\beta z+1}$	$C = \tan[0.5(\Omega_2 - \Omega_1)]$, $\beta = \cos\Omega_0$ or $\beta = \frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$

Table 6: Windows for FIR filter design.

Window	**Expression** $w_N[n]$, $-0.5(N-1) \leq n \leq 0.5(N-1)$
Boxcar	1
Cosine	$\cos\left(\frac{n\pi}{N-1}\right)$
Riemann	$\text{sinc}^L\left(\frac{2n}{N-1}\right), L > 0$
Bartlett	$1-\frac{2\lvert n\rvert}{N-1}$
Von Hann (Hanning)	$0.5+0.5\cos\left(\frac{2n\pi}{N-1}\right)$
Hamming	$0.54+0.46\cos\left(\frac{2n\pi}{N-1}\right)$
Blackman	$0.42+0.5\cos\left(\frac{2n\pi}{N-1}\right)+0.08\cos\left(\frac{4n\pi}{N-1}\right)$
Kaiser	$\frac{I_0\left(\pi\beta\sqrt{1-4\left(\frac{n}{N-1}\right)^2}\right)}{I_0(\pi\beta)}$

Table 7: Characteristics of the windowed spectrum for various windows.

Window	Peak Ripple $\delta_p = \delta_s$	Passband Attenuation A_{WP}(dB)	Peak Sidelobe Attenuation A_{WS} (dB)	Transition Width $F_{WS} \approx C/N$
Boxcar	0.0897	1.5618	21.7	$C = 0.92$
Cosine	0.0207	0.3600	33.8	$C = 2.10$
Riemann	0.0120	0.2087	38.5	$C = 2.50$
von Hann (Hanning)	0.0063	0.1103	44.0	$C = 3.21$
Hamming	0.0022	0.0384	53.0	$C = 3.47$
Blackman	1.71×10^{-4}	2.97×10^{-3}	75.3	$C = 5.71$

Euler Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$j^2 = -1 , \qquad e^{\pm\frac{j\pi}{2}} = \pm j , \qquad e^{\pm jk\pi} = cos(k\pi)$$

Finite Summation Formula

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha} , \quad \alpha \neq 1$$

$$\sum_{k=0}^{n} k\alpha^k = \frac{\alpha\left[1-(n+1)\alpha^n + n\alpha^{n+1}\right]}{(1-\alpha)^2}$$

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$$\sum_{k=0}^{n} k^2\alpha^k = \frac{\alpha\left[(1+\alpha)-(n+1)^2\alpha^n + \left(2n^2+2n-1\right)\alpha^{n+1} - n^2\alpha^{n+2}\right]}{(1-\alpha)^3}$$

Infinite Summation Formula

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} \alpha^k = \frac{\alpha}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k\alpha^k = \frac{\alpha}{(1-\alpha)^2}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k^2\alpha^k = \frac{\alpha^2+\alpha}{(1-\alpha)^3}, \qquad |\alpha| < 1$$

$$\sum_{k=-\infty}^{\infty} e^{-\alpha|k|} = \frac{1+e^{-\alpha}}{1-e^{-\alpha}}, \qquad \alpha > 0$$