Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS / ENGINEERING MATHEMATIC II
COURSE CODE	:	BEE11203 / BEE11403
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

Q1 (a) Given the differential equation of

$$\frac{dy}{dx} = 3y - x^2y$$

(i) Show that the equation can be solved using separation of variables method.

(3 marks)

(ii) Find the general solution for y using separation of variables method.

(3 marks)

(iii) Find the particular solution for y, given $y(0) = 2$.

(3 marks)

(b) Check the exactness of the differential equation

$$\frac{1}{x}dy = \frac{y}{x^2}dx$$

(4 marks)

(c) Given

$$\frac{dy}{dx} = \frac{-(x+y)^2}{2xy + x^2 - 1}$$

Find the particular solution for the differential equation using the exact method. The initial condition is given as $y(1) = 2$.

(12 marks)

Q2 (a) Given a second-order differential equation (DE) function of

$$\frac{d^2y}{dx^2} = y + xe^x$$

(i) Identify the complementary function, y_c for the corresponding homogeneous equation.

(2 marks)

(ii) Solve the given second-order DE function using method of undetermined coefficient with initial conditions $y(0) = 1$ and $y'(0) = 0$.

(11 marks)

(b) By using variation of parameter method, solve the differential equation below. (Hint: $cosh(ax) = \frac{1}{2}(e^{ax} + e^{-ax})$)

$$y'' - y = 6\cosh x$$

(12 marks)

Q3 (a) A system of first-order DE is given as

$$y_1' = y_1 + 6y_2 + 2e^{-x}$$

$$y_2' = 3y_1 + 8y_2 - 4e^{-x}$$

(i) Evaluate the eigenvalues for the above homogeneous system.
(3 marks)

(ii) Find the corresponding eigenvectors for the above homogeneous system.
(6 marks)

(iii) Formulate the general solution for the homogenous system.
(1 mark)

(b) Calculate the particular integral, $\boldsymbol{Y_P}$, for the non-homogeneous system.
(10 marks)

(c) Determine the general solution, $\boldsymbol{Y}$, for the non-homogeneous system.
(1 mark)

(d) Obtain the particular solution for $y_1(x)$ and $y_2(x)$ with $\boldsymbol{Y}(0) = \binom{1}{1}$.
(4 marks)

Q4 (a) Find the Laplace transform of the following functions.

(i) $\mathcal{L}\left\{e^{-3t}\cosh 5t\right\}$
(2 marks)

(ii) $\mathcal{L}\left\{t^2 \sin 3t\right\}$
(5 marks)

(b) Evaluate $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+2s+9}\right\}$
(4 marks)

(c) Compute $\mathcal{L}^{-1}\left\{\dfrac{2}{s(s^2+5)}\right\}$ using

(i) Transform of integral
(3 marks)

(ii) Partial fraction

(4 marks)

(iii) Convolution theorem

(4 marks)

(d) Find $\mathcal{L}^{-1}\left\{e^{-s}\left(\frac{2}{s^3}+\frac{2}{s^2}+\frac{2}{s}\right)\right\}$

(3 marks)

- END OF QUESTIONS -

APPENDIX A

First Order Exact Differential Equation

$$M(x,y)dx + N(x,y)dy = 0$$

$$I(x,y) = \int M\,dx + f(y)$$

$$I(x,y) = \int N\,dy + g(x)$$

Second-Order Liner Constant Coefficient Homogeneous Differential Equation

The roots of characteristic equation and the general solution for differential equation

$$ay''(x) + by'(x) + cy(x) = 0$$

Characteristic equation: $am^2 + bm + c = 0.$		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i, m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta\, x + B\sin\beta\, x)$

Second-Order Linear Constant Coefficient Non-Homogeneous Differential Equation

$$ay''(x) + by'(x) + cy(x) = f(x)$$

Variation of Parameter

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	
$u = -\int \frac{y_2 f(x)}{aW} dx + A$	$v = \int \frac{y_1 f(x)}{aW} dx + B$
$y = uy_1 + vy_2$	

APPENDIX B

The Method Of Undetermined Coefficient For Second-Order Linear Constant Coefficient Non-Homogeneous Differential Equation

$$ay''(x) + by'(x) + cy(x) = f(x)$$

General solution is $y = y_c + y_p$

Particular Integral, y_p based on $f(x)$

Type of $f(x)$	Example of $f(x)$	Assumption of y_p
Exponent	ke^{mx}	Ce^{mx}
Polynomial	k	C
	kx	$Cx+D$
	kx^2	Cx^2+Dx+E
	$kP_n(x)$	$C_nx^n + C_{n-1}x^{n-1} + \ldots + C_1x + C_0$
Trigonometry (sin and cos only)	$k\sin px$ or $k\cos px$	$C\cos px + D\sin px$
	$k\sinh px$ or $k\cosh px$	$C\cosh px + D\sinh px$
Product of polynomial and exponential	$P_n(x)e^{mx}$	$(C_nx^n + C_{n-1}x^{n-1} + \ldots + C_1x + C_0)e^{mx}$
Product of polynomial and trigonometry	$P_n(x)\sin px$ or $P_n(x)\cos px$	$(C_nx^n + C_{n-1}x^{n-1} + \ldots + C_1x + C_0)\sin px +$ $(D_nx^n + D_{n-1}x^{n-1} + \ldots + D_1x + D_0)\cos px$
Product of exponential and trigonometry	$ke^{mx}\sin px$	$e^{mx}(C\cos px + D\sin px)$

APPENDIX C

Homogeneous System Of First-Order Differential Equation

$$Y'(x) = AY(x)$$

Eigenvalues
$|A-\lambda\mathbf{I}|=0$

Eigenvectors
$(A-\lambda\mathbf{I})\mathbf{V}=0$

Case	Roots	General solution
1	Real and Distinct eigenvalues	$\mathbf{Y}=A\mathbf{V}_1e^{\lambda_1 x}+B\mathbf{V}_2e^{\lambda_2 x}$
2	Repeated eigenvalues	$\mathbf{Y}=A\mathbf{V}_1e^{\lambda x}+B\left[\mathbf{V}_1x+\mathbf{V}_2\right]e^{\lambda x}$

Nonhomogeneous System Of First Order Linear Differential Equations

$$Y'(x) = AY(x) + G(x)$$

General solution is $Y = Y_C + Y_P$

Particular Integral, Y_p based on G

Assume $\mathbf{Y}_P$ based on $\mathbf{G}$		
Case	$\mathbf{G}$	$\mathbf{Y}_P$
Case I	Polynomial $\begin{pmatrix}a_1\\a_2\end{pmatrix};\begin{pmatrix}a_1x+b_1\\a_2x+b_2\end{pmatrix};$ $\begin{pmatrix}a_1x^2+b_1x+c_1\\a_2x^2+b_2x+c_2\end{pmatrix}$	$\begin{pmatrix}C\\D\end{pmatrix};\begin{pmatrix}Cx+E\\Dx+F\end{pmatrix};$ $\begin{pmatrix}Cx^2+Ex+G\\Dx^2+Fx+H\end{pmatrix}$
Case II	Exponent $\begin{pmatrix}a_1\\a_2\end{pmatrix}e^{kx}$	$\begin{pmatrix}C\\D\end{pmatrix}e^{kx}$ if $\mathbf{Y}_P\equiv\mathbf{Y}_C$, then $\begin{pmatrix}C\\D\end{pmatrix}xe^{kx}+\begin{pmatrix}E\\F\end{pmatrix}e^{kx}$
Case III	Trigonometric (sin and cos only) $\begin{pmatrix}a_1\\a_2\end{pmatrix}\sin kx$ or $\begin{pmatrix}a_1\\a_2\end{pmatrix}\cos kx$	$\begin{pmatrix}C\\D\end{pmatrix}\sin kx+\begin{pmatrix}E\\F\end{pmatrix}\cos kx$

APPENDIX D

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s) \text{ for } t \geq 0$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
e^{at}	$\frac{1}{s-a}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\sinh at$	$\frac{a}{s^2-a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y(t)$	$Y(s)$
$e^{at}f(t)$	$F(s-a)$	$y'(t)$	$sY(s)-y(0)$
$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

Partial Fraction

Linear factors, $\frac{as+b}{(s+c)(s+d)} = \frac{A}{s+c} + \frac{B}{s+d}$

Repeated linear factors,

$$\frac{as+b}{(s+c)^n} = \frac{A_1}{s+c} + \frac{A_2}{(s+c)^2} + \frac{A_3}{(s+c)^3} + \cdots \frac{A_n}{(s+c)^n}$$

APPENDIX E

TRIGONOMETRIC SUBSTITUTION		
Expression	***Trigonometry***	***Hyperbolic***
$\sqrt{x^2+k^2}$	$x=k\tan\theta$	$x=k\sinh\theta$
$\sqrt{x^2-k^2}$	$x=k\sec\theta$	$x=k\cosh\theta$
$\sqrt{k^2-x^2}$	$x=k\sin\theta$	$x=k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION			
$t=\tan\frac{1}{2}x$		$t=\tan x$	
$\sin x=\dfrac{2t}{1+t^2}$ $\tan x=\dfrac{2t}{1-t^2}$	$\cos x=\dfrac{1-t^2}{1+t^2}$ $dx=\dfrac{2dt}{1+t^2}$	$\sin 2x=\dfrac{2t}{1+t^2}$ $\tan 2x=\dfrac{2t}{1-t^2}$	$\cos 2x=\dfrac{1-t^2}{1+t^2}$ $dx=\dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC	
Trigonometric Functions	***Hyperbolic Functions***
$\cos^2 x+\sin^2 x=1$ $\sin 2x=2\sin x\cos x$ $\cos 2x=\cos^2 x-\sin^2 x$ $=2\cos^2 x-1$ $=1-2\sin^2 x$ $1+\tan^2 x=\sec^2 x$ $1+\cot^2 x=\csc^2 x$ $\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$ $\tan(x\pm y)=\dfrac{\tan x\pm\tan y}{1\mp\tan x\tan y}$ $\sin(x\pm y)=\sin x\cos y\pm\sin y\cos x$ $\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$ $2\sin ax\cos bx=\sin(a+b)x+\sin(a-b)x$ $2\sin ax\sin bx=\cos(a-b)x-\cos(a+b)x$ $2\cos ax\cos bx=\cos(a-b)x+\cos(a+b)x$	$\sinh x=\dfrac{e^x-e^{-x}}{2}$ $\cosh x=\dfrac{e^x+e^{-x}}{2}$ $\cosh^2 x-\sinh^2 x=1$ $\sinh 2x=2\sinh x\cosh x$ $\cosh 2x=\cosh^2 x+\sinh^2 x$ $=2\cosh^2 x-1$ $=1+2\sinh^2 x$ $1-\tanh^2 x=\text{sech}^2 x$ $\coth^2 x-1=\text{csch}^2 x$ $\tanh 2x=\dfrac{2\tanh x}{1+\tanh^2 x}$ $\tanh(x\pm y)=\dfrac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$ $\sinh(x\pm y)=\sinh x\cosh y\pm\sinh y\cosh x$ $\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$

APPENDIX F

Indefinite Integrals	Integration of Inverse Functions
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{1}{x}\, dx = \ln\|x\| + C$	$\int \frac{-1}{\sqrt{a^2 - x^2}}\, dx = \cos^{-1}\left(\frac{x}{a}\right) + C$
$\int \cos x\, dx = \sin x + C$	$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \frac{-1}{a^2 + x^2}\, dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$
$\int \sec^2 x\, dx = \tan x + C$	$\int \frac{1}{\|x\|\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc^2 x\, dx = -\cot x + C$	$\int \frac{-1}{\|x\|\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C$
$\int \sec x \tan x\, dx = \sec x + C$	$\int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc x \cot x\, dx = -\csc x + C$	$\int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \cosh^{-1}\left(\frac{x}{a}\right) + C$
$\int e^x dx = e^x + C$	$\int \frac{-1}{\|x\|\sqrt{a^2 - x^2}}\, dx = \frac{1}{a}\operatorname{sech}^{-1}\left\|\frac{x}{a}\right\| + C$
$\int \cosh x\, dx = \sinh x + C$	$\int \frac{-1}{\|x\|\sqrt{a^2 + x^2}}\, dx = \frac{1}{a}\operatorname{csch}^{-1}\left\|\frac{x}{a}\right\| + C$
$\int \sinh x\, dx = \cosh x + C$	$\int \frac{1}{a^2 - x^2}\, dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \|x\| < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \|x\| > a \end{cases}$
$\int \operatorname{sech}^2 x\, dx = \tanh x + C$	
$\int \operatorname{csch}^2 x\, dx = -\coth x + C$	
$\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x + C$	
$\int \operatorname{csch} x \coth x\, dx = -\operatorname{csch} x + C$	