Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	NUMERICAL METHODS / ENGINEERING MATHEMATICS IV
COURSE CODE	:	BEE32402/ BEE31602
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JULY 2024
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) The velocity of a rocket is given by:

$$v(t) = 2000ln\left[\frac{14\times10^4}{14\times10^4 - 2100t}\right] - 9.8t, \qquad 0 \le t \le 30$$

Where v is given in m/s and t is given in seconds.

(i) By taking $h = 2s$, approximate first derivative of $v(t)$ to calculate the acceleration at $t = 16s$ using 2-point forward, 2-point backward and 3-point central differential approximation.

(11 marks)

(ii) Find the exact value of the acceleration of the rocket using a scientific calculator.

(2 marks)

(iii) Calculate the absolute error for each method from **Q1(a).**

(3 marks)

(iv) Identify the best method in approximating the acceleration of the rocket.

(1 mark)

(b) A point A is moving along the curve whose equations is $f(x) = 2x^2 + \sin(x)$. By using second derivatives for 3-point central and 5-point difference formula with $h = 0.1$. Calculate how far A is moving when $x = 1.2$.

(8 marks)

Q2 According to Kirchhoff's voltage law, a simple series RL circuit that can consist of a resistor, an inductor and a power supply can be represented by the following equation.

$$L\frac{di}{dt} + Ri = E(t)$$

Given $E(t) = 120\ V$, L = 3 H, R=15 Ω, $i = 5.0570\ A$ when $t = 0.20\ s$

(a) Calculate the $i(t)$ between $0.20\ s$ and $0.25\ s$ with an interval of $0.01\ s$ using Euler's method.

(7 marks)

(b) Given $i = 5.7080A$ when $t = 0.25\ s$, calculate the $i(t)$ between $0.20\ s$ and $0.25\ s$ with an interval of 0.01s using finite-different method.

(12 marks)

(c) Investigate the absolute errors at each estimation at the **Q2(a)** and **Q2(b)** if the exact solution is $i(t) = 8(1 - e^{-5t})$.

(6 marks)

Q3 The graph for $\int_1^4 \frac{x-2}{\sqrt{x}}\, dx$ is shown in **Figure Q3**.

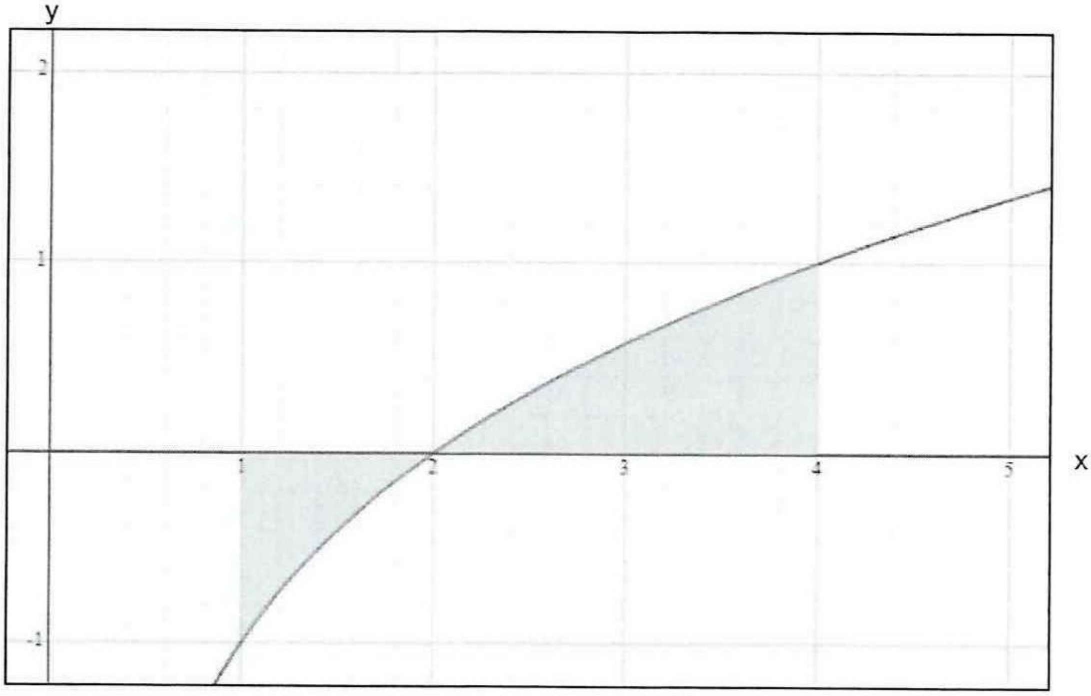

Figure Q3: Graph

(a) Approximate the total volume of the shaded area with subintervals of $n = 10$ and $n = 15$ by using:

(i) Trapezoidal rule.

(12 marks)

(ii) Simpson's 1/3 rule (with appropriate subintervals).

(8 mark)

(b) Calculate the exact solution by using a scientific calculator.

(1 mark)

(c) Find the absolute errors for each of the methods in **Q3(a)**.

(3 marks)

(d) Determine which method gives the better approximation.

(1 mark)

Q4 (a) The temperature distribution $u(x, t)$ of a one-dimensional silver rod is governed by the heat equation as follows.

$$\frac{\partial u}{\partial t} = 0.5\frac{\partial^2 u}{\partial x^2}$$

Given the boundary conditions $u(0, t) = t^2$, $u(0.6, t) = 6t + 0.12$, for $0 \leq t \leq 0.04s$ and the initial condition $u(x, 0) = x(0.8 - x)$ for $0 \leq x \leq 0.6$mm, analyze the temperature distribution of the rod with $\Delta x = 0.2$mm and $\Delta t = 0.02$s using explicit method (i.e. Forward Time Central Space (FTCS) finite-difference approximation) in 4 decimal places.

(12 marks)

(b) An elastic string is fixed at both ends and is governed by the wave equation as follows.

$$\frac{\partial^2 u}{\partial t^2} = 0.25\frac{\partial^2 u}{\partial x^2}$$

Where $u(x, t)$ of the displacement of the string with $0 \leq x \leq 1$ and $0 \leq \mathrm{t} \leq 0.4$. The initial and boundary conditions are as follows.

$$u(x, 0) = \sin(\pi x)$$

$$\frac{\partial u}{\partial t}(x, 0) = 0.25$$

Analyze the displacement of the string using the central time central space (CTCS) explicit finite-difference approximation with $\Delta x = 0.25$cm and $\Delta t = 0.2$s in 4 decimal places.

(13 marks)

-END OF QUESTIONS –

FORMULAE

First Order Numerical differentiation:

2-point forward difference

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

2-point backward difference

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

3-point central difference

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

3-point forward difference

$$f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$$

3-point backward difference

$$f'(x) \approx \frac{f(x-2h) - 4f(x-h) + 3f(x)}{2h}$$

5-point central difference

$$f'(x) \approx \frac{f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h))}{12h}$$

Second Order Numerical differentiation:

3-point central difference formula (second derivative)

$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

5-point formula for second derivative

$$f'(x) \approx \frac{-f(x-2h) + 16f(x-h) - 30f(x) + 16f(x+h) - f(x+2h))}{12h^2}$$

Euler method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

Boundary value problems:

Finite difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h} \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Numerical Integration:

Trapezoidal rule:

$$\int_a^b f(x)d(x) \approx \frac{h}{2}\left[f_0 + f_n + 2\sum_{i=1}^{n-1} f_i\right]$$

Simpson's $\frac{1}{3}$ rule:

$$\int_a^b f(x)d(x) \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{i=1}^{n/2} f_{2i-1} + 2\sum_{i=1}^{(n/2)-1} f_{2i}\right]$$

Simpson's $\frac{3}{8}$ rule:

$$\int_a^b f(x)d(x) \approx \frac{3h}{8}\left[f_0 + f_n + 3\sum_{i=1}^{n/3}(f_{3i-2} + f_{3i-1}) + 2\sum_{i=1}^{(n/3)-1} f_{3i}\right]$$