UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	ROBOTIC SYSTEMS
COURSE CODE	:	BEJ44203
PROGRAMME CODE	:	BEJ
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) What is the RIA definition of a robot?

(3 marks)

(b) Draw the approximate workspace for the robot as shown in **Figure Q1.1.**

(3 marks)

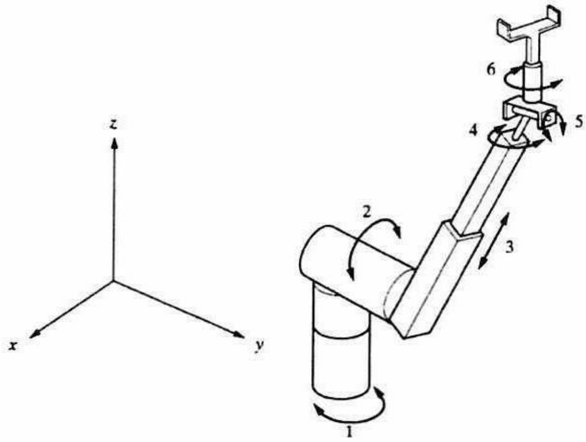


Figure Q1.1: Robot configuration.

(c) State **THREE (3)** advantages and **THREE (3)** disadvantages of the robot s shown in **Figure Q1.1.**

(6 marks)

(d) A frame has been moved 5 units along the x-axis, 5 units along the y-axis and 5 units along the Z-axis of the reference frame. Determine the new location of the frame:

$$F = \begin{bmatrix} 0.7 & -0.4 & 0.8 & 5 \\ 0.9 & 0.9 & 0.9 & 3 \\ -0.7 & 0 & 0.6 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2 marks)

(e) A point P= $[7, 3, 2]^T$, attached to a frame (n, o, a) relative to frame B, is subjected to the following transformation. Calculate the total transformation matrix:

(i) A rotation of 45° about the y-axis,

(1.5 marks)

(ii) Followed by a rotation of 45° about the o-axis,

(1.5 marks)

(iii) Followed by a translation of 4 units along the n-axis,

(1.5 marks)

(iv) Followed by a translation of 4 units along the x-axis.

(1.5 marks)

Q2 For the SCARA-type robot shown in **Figure Q2.1**:

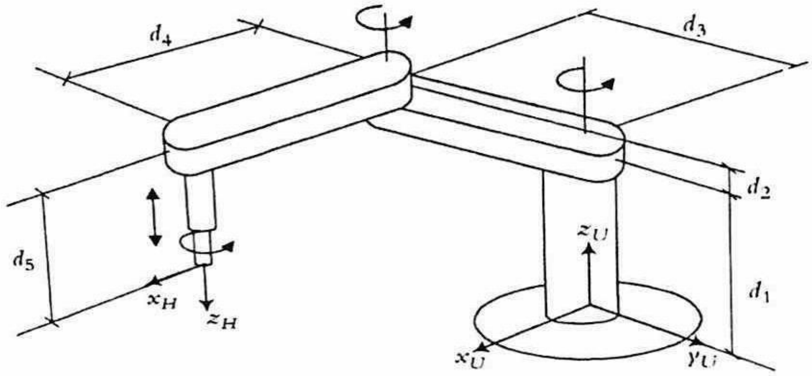


Figure Q2.1: SCARA Robot

(a) Assign the coordinate frames based on D-H representation.

(6 marks)

(b) Fill out the parameters table.

(10 marks)

(c) Write the ${}^{U}\boldsymbol{T}_{H}$ matrix in terms of the A matrices.

(4 marks)

Q3 (a) Suppose that a robot is made of a Cartesian and RPY combination of joints. Calculate the necessary RPY angles to achieve the following:

$$B = \begin{bmatrix} 0.354 & 0.674 & 0.649 & 4.33 \\ 0.505 & 0.722 & 0.475 & 2.5 \\ 0.788 & 0.160 & 0.595 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(8 marks)

(b) Calculate the inverse of the following transformation matrix:

$$F = \begin{bmatrix} 0.5 & 0 & 0.866 & 3 \\ 0.866 & 0 & 5 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3 marks)

(c) It is desired to have the first joint of a 6-axis robot go from initial angle of 50° to a final angle of 80° in 3 seconds. Using a third-order polynomial, calculate the joint angle at 1, 2, 3, and 4 seconds. Produce the graph of joint positions, velocities, and accelerations of the trajectory planning.

(9 marks)

Q4 (a) The position of point *B* of **Figure Q4.1** is as follows:

$$x_B = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$
$$y_B = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$$

Calculate the differential motion of *B*, its Jacobian and its differential of joints in matrix form.

(5 marks)

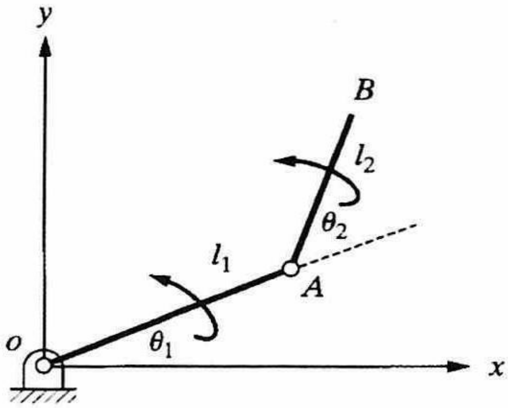


Figure Q4.1: Planar manipulator with massless links

(b) The last column of the forward kinematic equation of the simple revolute arm is:

$$\begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = \begin{bmatrix} C_1(C_{234}a_4 + C_{23}a_3 + C_2a_2) \\ S1(C_{234}a_4 + C_{23}a_3 + C_2a_2) \\ S_{234}a_4 + S_{23}a_3 + S_2a_2 \\ 1 \end{bmatrix}$$

Calculate the Jacobian of P_y of the above revolute arm robot.

(6 marks)

(c) The last column of the forward kinematic equation of a simple revolute arm is:

$$A1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A2 = \begin{bmatrix} C_2 & -S_2 & 0 & C_2a_2 \\ S_2 & C_2 & 0 & S_2a_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A3 = \begin{bmatrix} C_3 & -S_3 & 0 & C_3a_3 \\ S_3 & C_3 & 0 & S_3a_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A4 = \begin{bmatrix} C_4 & 0 & -S_4 & C_4a_4 \\ S_4 & 0 & C_4 & S_4a_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Calculate the ${}^{T6}J_{24}$ element of the Jacobian for the above revolute arm robot.

(9 marks)

Q5 (a) Consider a planar manipulator with massless links and two massive points m_1 and m_2 as shown in **Figure Q5.1**. Formulate the equation of motion.

(12 marks)

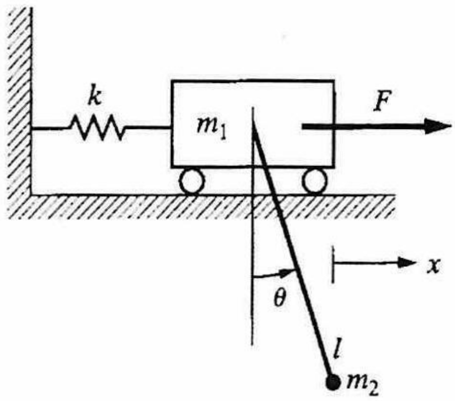


Figure Q5.1: Cart pendulum system.

(b) (i) Discuss the concept of Cartesian control in manipulator systems. Explain how Cartesian control differs from joint-space control and what advantages it offers.

(2 marks)

(ii) Discuss the concept of hybrid position/force control in manipulator systems. Explain the key components and principles of hybrid position/force controllers and provide insights into their implementation challenges and advantages. Illustrate your discussion with examples of tasks where hybrid control is beneficial.

(6 marks)

- END OF QUESTION –

APPENDIX A

FORMULAE

New location of frame due to the pure translation

$F_{new} = Trans(d_x, d_y, d_z)\ x\ F_{old}$

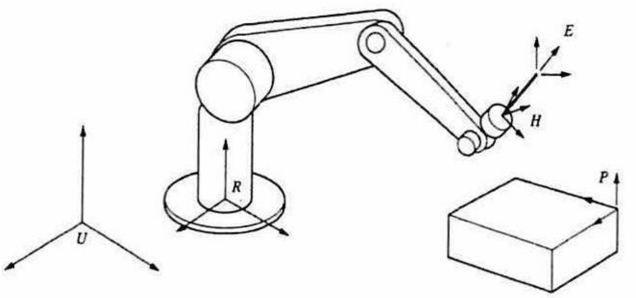
Six constraint equations:

(1) $\bar{n} \bullet \bar{o} = 0$
(2) $\bar{n} \bullet \bar{a} = 0$
(3) $\bar{a} \bullet \bar{o} = 0$
(4) $|n| = 1$
(5) $|o| = 1$
(6) $|a| = 1$

Rotation portion of the matrix

$$T^{-1} = \begin{bmatrix} n_x & n_y & n_z & -\bar{P}.\bar{n} \\ o_x & o_y & o_z & -\bar{P}.\bar{o} \\ a_x & a_y & a_z & -\bar{P}.\bar{a} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation from reference frame U to end effector:



$${}^{U}T_{E} = {}^{U}T_{R}\,{}^{R}T_{H}\,{}^{H}T_{E} = {}^{U}T_{P}\,{}^{P}T_{E}, \qquad {}^{R}T_{H} = {}^{U}T_{R}^{-1}\,{}^{U}T_{P}\,{}^{P}T_{E}\,{}^{H}T_{E}^{-1}$$

- ${}^{U}T_{R}$ is the transformation of frame R relative to U.
- ${}^{H}T_{E}$ is the transformation of the end effector relative to robot's hand.
- ${}^{U}T_{P}$ is the transformation of the part relative to the universe.
- ${}^{P}T_{E}$ is the transformation of the end effector relative to the part's position.
- ${}^{R}T_{H}$ is the transformation of the robot's hand relative to the robot's base (unknown).

Representation of the rotation matrix

$$Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}, \quad Rot(y,\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}, \quad Rot(z,\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Cylindrical Coordinates

$${}^{R}T_{P} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\alpha & -S\alpha & 0 & rC\alpha \\ S\alpha & C\alpha & 0 & rS\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spherical Coordinates

$${}^{R}T_{P} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\beta C\gamma & -S\gamma & S\beta C\gamma & rS\beta C\gamma \\ C\beta S\gamma & C\gamma & S\beta S\gamma & rS\beta S\gamma \\ -S\beta & C\beta C\gamma & C\beta & rC\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

FORMULAE

Roll, Pitch, Yaw (RPY) Angles

$\Phi_a = ATAN2(n_y, n_x)$, $\Phi_o = ATAN2(-n_z, (n_x C\Phi_a + n_y S\Phi_a))$,

$\Phi_n = ATAN2((-a_y C\Phi_a - a_x S\Phi_a), (-o_y C\Phi_a - o_x S\Phi_a))$

Eulers Angles

$\Phi = ATAN2(a_y, a_x)$, $\Psi = ATAN2(-n_x S\Phi + n_y C\Phi, -o_x S\Phi + o_y C\Phi)$,

$\theta = ATAN2(a_x C\Phi + a_y S\Phi), a_z)$

Denavit-Hartenberg Representation

- θ represents the rotations about the z-axis.
- d represents the distance on the z-axis between two successive common normals.
- a represents the length of each common normal (also called joint offset).
- α represents the angles between two successive z-axis (also called joint twist)

Representation of A matrices

$$^{n}T_{n+1} = A_{n+1} = Rot(z, \theta_{n+1}) Tran(0,0,d_{n+1}) Trans(a_{n+1},0,0)(Rot(x, \alpha_{n+1})$$

$$= \begin{bmatrix} C\theta_{n+1} & -S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Jacobian Matrix

$$D = JD\theta$$

$$\begin{bmatrix} dx \\ dy \\ dz \\ \delta x \\ \delta y \\ \delta z \end{bmatrix} = \begin{bmatrix} \\ Robot \\ Jacobian \\ \\ \end{bmatrix} \begin{bmatrix} d\theta 1 \\ d\theta 2 \\ d\theta 3 \\ d\theta 4 \\ d\theta 5 \\ d\theta 6 \end{bmatrix}$$

The Jacobian with respect to the last frame

$$\begin{bmatrix} ^{T6}dx \\ ^{T6}dy \\ ^{T6}dz \\ ^{T6}\delta x \\ ^{T6}\delta y \\ ^{T6}\delta z \end{bmatrix} = \begin{bmatrix} ^{T6}J_{11} & ^{T6}J_{12} & ^{T6}J_{13} & ^{T6}J_{14} & ^{T6}J_{15} & ^{T6}J_{16} \\ ^{T6}J_{21} & ^{T6}J_{22} & ^{T6}J_{23} & ^{T6}J_{24} & ^{T6}J_{25} & ^{T6}J_{26} \\ ^{T6}J_{31} & ^{T6}J_{32} & ^{T6}J_{33} & ^{T6}J_{34} & ^{T6}J_{35} & ^{T6}J_{36} \\ ^{T6}J_{41} & ^{T6}J_{42} & ^{T6}J_{43} & ^{T6}J_{44} & ^{T6}J_{45} & ^{T6}J_{46} \\ ^{T6}J_{51} & ^{T6}J_{52} & ^{T6}J_{53} & ^{T6}J_{54} & ^{T6}J_{55} & ^{T6}J_{56} \\ ^{T6}J_{61} & ^{T6}J_{62} & ^{T6}J_{63} & ^{T6}J_{64} & ^{T6}J_{65} & ^{T6}J_{66} \end{bmatrix} \begin{bmatrix} d\theta 1 \\ d\theta 2 \\ d\theta 3 \\ d\theta 4 \\ d\theta 5 \\ d\theta 6 \end{bmatrix}$$

- **If i under consideration is a revolute joint, then:**

  $^{T6}J_{1i} = (-n_x p_y + n_y p_x)$, $^{T6}J_{2i} = (-o_x p_y + o_y p_x)$, $^{T6}J_{3i} = (-a_x p_y + a_y p_x)$

  $^{T6}J_{4i} = n_z$, $^{T6}J_{5i} = o_z$, $^{T6}J_{6i} = a_z$

- **Assuming that any combination of $A_1A_2A_3A_4A_5A_6$ can be expressed with a corresponding n , o , a , p matrix, the corresponding elements of the matrix will be used to calculate the Jacobian.**

FORMULAE

- **If *i* under consideration is a prismatic joint, then:**

  $^{T6}J_{1i} = n_z,\ ^{T6}J_{2i} = o_z,,\ ^{T6}J_{3i} = a_z,\ ^{T6}J_{4i} = 0,\ ^{T6}J_{5i} = 0,\ ^{T6}J_{6i} = 0$

- **For column *i* use $^{i-1}T_6$:**

  For column 1, use $^0T_6 = A_1A_2A_3A_4A_5A_6$

  For column 2, use $^1T_6 = A_2A_3A_4A_5A_6$

  For column 3, use $^2T_6 = A_3A_4A_5A_6$

  For column 4, use $^3T_6 = A_4A_5A_6$

  For column 5, use $^4T_6 = A_5A_6$

  For column 6, use $^5T_6 = A_6$

Differential transformation

$$dT = [\Delta][T]$$

Differential operator relative to the fixed frame

$$\Delta = [Trans(dx, dy, dz)Rot(k, d\theta) - I] = \begin{bmatrix} 0 & -\delta z & \delta y & dx \\ \delta z & 0 & -\delta x & dy \\ -\delta y & \delta x & 0 & dz \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Differential operator relative to the current frame

$$[^T\Delta] = [T]^{-1}[\Delta][T] = \begin{bmatrix} 0 & -^T\delta z & ^T\delta y & ^T dx \\ ^T\delta z & 0 & -^T\delta x & ^T dy \\ -^T\delta y & ^T\delta x & 0 & ^T dz \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

New location and orientation of the frame

$$B_{new} = B_{original} + dB$$

Lagrange function

$$L = K(q, \dot{q}) - P(q)$$

Total kinetic energy

$$K = \sum_{i=1}^{n} \frac{m1(\dot{x}_t^2 + \dot{y}_t^2 + \dot{z}_t^2)}{2}$$

Potential energy

$$P = mgh$$

General Lagrangian equation for the *i*th particle

$$F_i = \frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}_i}\right) - \left(\frac{\delta L}{\delta x_i}\right), i = 1,2\ldots\ldots n$$

$$T_i = \frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}_i}\right) - \left(\frac{\delta L}{\delta \theta_i}\right), i = 1,2\ldots\ldots n$$

3rd-order polynomial:

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$

$$\ddot{\theta}(t) = 2c_2 + 6c_3 t$$