Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	DISCRETE STRUCTURE
COURSE CODE	:	BIT 11003
PROGRAMME CODE	:	BIT
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 (a) Analyse whether $\neg(p \oplus q) \leftrightarrow (\neg q \vee r)$ is tautology, contradiction or contingency by constructing the truth table.

(8 marks)

(b) Let p, q and r be the propositions of "The user enters a valid password", "Access is granted" and "The user has paid the subscription fee", respectively. Translate the following English sentences into logical expressions as precisely as possible.

(i) The user has paid the subscription fee, but does not enter a valid password.

(1 mark)

(ii) If the user has not entered a valid password but has paid the subscription fee, then access is granted.

(1 mark)

(iii) Access is granted whenever the user has paid the subscription fee and enters a valid password.

(1 mark)

(iv) It is neither the user enters a valid password nor has paid the subscription fee.

(1 mark)

(c) Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent or not.

(8 marks)

Q2 (a) Define a sequence $a_1, a_2, a_3, \ldots\ldots a_k$ as follows:

$$a_1 = 0$$
$$a_2 = 2$$
$$a_k = 3a_{k-1} + 2 \text{ for each integer } k \geq 3$$

Find the first **SIX (6)** terms of the sequence.

(5 marks)

(b) Use mathematical induction to prove that $3^{2n} - 1$ is divisible by 8 for all natural numbers.

(10 marks)

Q3 Answer **Q3(a)** to **Q3(c)** according to **Figure Q3.1**.

Suppose A is the set of distinct letters in the word 'elephant', B is the set of distinct letters in the word 'sycophant' and C is the set of distinct letters in the word 'fantastic'. The universe U is the set of 26 lower-case letters of the English alphabet.

Figure Q3.1

Find the following;

(a) $A \cup C$.

(3 marks)

(b) $C \cap B$.

(3 marks)

(c) $(A \cup B \cup C)'$.

(4 marks)

Q4 (a) Answer **Q4(a)(i)** and **Q4(a)(ii)** according to **Figure Q4.1**.

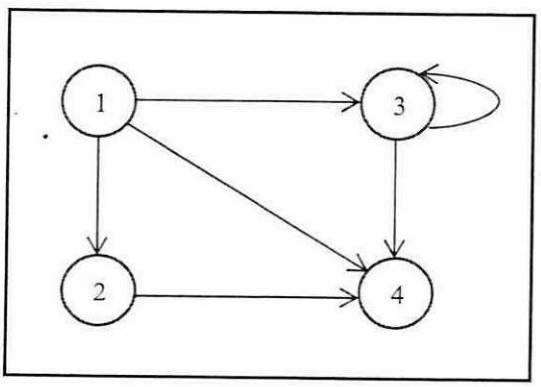

Figure Q4.1

(i) Find the domain and range of the relation.

(4 marks)

(ii) Show this relation is transitive.

(4 marks)

(b) Given $f(x) = -3x$ and $g(x) = x^2 + 3$.

(i) Find the composite function of $(f \circ g)(x)$.

(3 marks)

(ii) Evaluate the function of $(g \circ f)(4)$.

(4 marks)

Q5 (a) Answer **Q5(a)(i)** to **Q5(a)(iii)** according to **Figure Q5.1**

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In the Faculty of Computer Science and Information Technology,
the enrolment of the IT Club has been steadily growing year by
year. Currently, there are 30 students actively in the Club.
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Figure Q5.1

(i) Write the recurrence relation formula if the number of students are increasing by two each year.

(1 mark)

(ii) Write down the solution formula of a_n in terms of n for the recurrence relation answered in **Q5(a)(i)**.

(2 marks)

(iii) Find the number of students expected to be enrolled in the club after three years. Show your work.

(3 marks)

(b) Formulate a solution to the following recurrence relations.

$$a_n - 4a_{n-1} = -4a_{n-2} \text{ where } a_0 = 1, a_1 = 3.$$

(9 marks)

Q6 Describe whether the graph in **Figure Q6.1** has an Hamilton cycle or Hamilton path. Based on your answer, list out the cycle or the path.

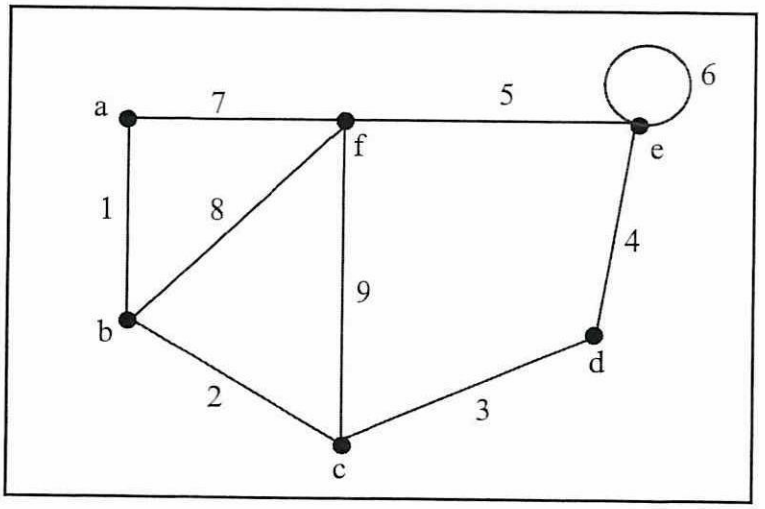

Figure Q6.1

(5 marks)

- END OF QUESTIONS -