Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	ALGEBRA
COURSE CODE	:	BIC 10303
PROGRAMME CODE	:	BIP / BIS / BIW / BIM
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 Answer the following questions.

(a) Simplify the following equation and write the answer in ascending powers of x

$$(2x^2 - x - 3)(1 + 2x - x^2)$$

(3 marks)

(b) Use the factor theorem to show that $(x + 3)$ is a factor of $x^3 + 5x^2 - 2x - 24$

(3 marks)

(c) Given the function is

$$f(x) = x^3 - 3x^2 + 6x - 40$$

(i) Show that $(x - 5)$ is not a factor of $f(x)$

(3 marks)

(ii) Find a linear factor of $f(x)$

(3 marks)

(d) Given the polynomial function is

$$f(x) = 3x^3 - 2x^2 - 12x + 8$$

(i) Show that $(x + 2)$ is a factor of $f(x)$ by using factor theorem

(4 marks)

(ii) Factorize the $f(x)$ completely

(4 marks)

Q2 Answer the following questions

(a) Solve the following arithmetic and geometric sequence problems

(i) Find the next three terms in the geometric sequence 4, 8, 16, 32.

(3 marks)

(ii) The first term of an arithmetic sequence is 8, and the common difference is 4. Find the 15^{th} term of the sequence.

(3 marks)

(iii) Find the 10^{th} term of the geometric sequence $3, 9, 27, 81, \ldots, n$

(2 marks)

(iv) Find the sum of the infinite geometric series $4 + 2 + 1 + 0.5 + \cdots + n$

(3 marks)

(b) Answer **Q(b)(i)** to **Q(b)(ii)** based on **Figure Q2.1**

> Osama starts his new job on an annual salary of \$18000. His contract promises a pay rise of \$1800 each subsequent year until his salary reaches \$36000. When the salary reaches \$36000, Osama will receive no more pay rises. Osama's salary first reaches the maximum salary of \$36000 in year N.

Figure Q2.1

(i) Determine the value of N.

(2 marks)

(ii) Find Osama's total salary earnings during the first N years of his employment.

(3 marks)

Q3 Answer the following questions.

(a) The matrices A, B, and C are given below in terms of the scalar constants $a, b, c,$ and d, by

$$A = \begin{bmatrix} -2 & 3 \\ 1 & a \end{bmatrix} \quad B = \begin{bmatrix} b & -1 \\ 2 & -4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & c \\ d & 4 \end{bmatrix}$$

Given that A + B = C, find the value of a, b, c and d.

(6 marks)

(b) Given a transformation $T : \mathbb{R}^2 \longmapsto \mathbb{R}^2$ is represented by the following 2 x2 matrix

$$A = \begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}$$

(i) Find the determinant of A.

(2 marks)

(ii) Find the equation of the straight line under the transformation represented by A.

(6 marks)

(c) Find $|A|$ of the matrix $A = \begin{bmatrix} -2 & 3 & 8 \\ 6 & 7 & -1 \\ -4 & 5 & 9 \end{bmatrix}$.

(3 marks)

(d) Given a matrix,

$$B = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

Show that $v = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ is the eigenvector that will fulfil the given eigenvalue if 8 is one of B's eigenvalues.

(3 marks)

Q4 (a) Use the graph solution to solve the following simultaneous equations.

(i) $x + y = 8$ and $2x + 3y = 21$

(6 marks)

(ii) $y = 8 - x$ and $y = 2x - 1$

(6 marks)

(b) Based on **Figure Q4.1**, calculate the cost per unit of labour and capital using the determinant method.

A commodity was produced using 3 units of labour and 2 units of capital; the total cost is \$62. If the commodity was produced using 4 units of labour and one unit of capital, the cost is \$56.

Figure Q4.1

(8 marks)

- END OF QUESTIONS -