UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	ENGINEERING MATHEMATICS
COURSE CODE	:	BFC 25103
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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Q1 (a) Find the Laplace Transform of this function $f(t) = e^{8t}\cos 5t$ using the First Shift theorem.

(5 marks)

(b) Determine the inverse Laplace Transform of $F(s) = \dfrac{1}{(s+4)(s-7)}$.

(10 marks)

(c) Determine the partial derivatives (f_{xy}, f_{yz} and f_{yx}) for the function $f(x,y,z) = y^3 zx\,ln(x)$.

(10 marks)

Q2 (a) Solve the following double integration:

$$\int_0^{\pi}\int_0^{x} x\sin y \; dy\, dx$$

(7 marks)

(b) By using double integrals, sketch and determine the area of the regions enclosed by $y = 8 - x^2$ and $y = 2x$.

(8 marks)

(c) Sketch the solid that enclosed above by hemisphere $z = \sqrt{25 - x^2 - y^2}$, below by plane $z = 2$ and side by cylinder $x^2 + y^2 = 9$. Sketch its projection on xy-plane. Find the moment of inertia about z-axis for the solid. Given the density function is $\rho\,(x, y, z) = z$.

(10 marks)

Q3 (a) Convert the following triple integral from Cartesian coordinates to cylindrical coordinates to solve:

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2} \sqrt{x^2+y^2}\; dz\, dy\, dx$$

(10 marks)

(b) Sketch the graph of the vector functions below:

(i) $r(t) = (t-2)i + (3-t)j + (3t+1)k$ where $-\infty < t < \infty$.

(3 marks)

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(ii) $r(t) = ti + t^2 j.$

(2 marks)

(c) In a civil engineering project, a cylindrical support column needs to be installed on a construction site. The column is represented by the equation $x^2 + z^2 = 4$ where x and z are the horizontal coordinates in meters, and the origin is at the center of the column's base. Simultaneously, a horizontal ground plane is situated at $y = 3$. This ground plane represents the level surface upon which the column will be erected.

(i) Determine the vector-valued function $r(t)$ that represents the path of intersection between the support column and the ground plane.

(6 marks)

(ii) Using the vector-valued function, r(t), sketch the graph of the intersection curve in the *xz*-plane, indicating the position of the column's base relative to the ground surface.

(4 marks)

Q4 (a) A pipeline with a diameter of 600 mm will be constructed in the upcoming development zone. The equation curve of the pipeline is given by $r(t) = \cos t\,\mathbf{i} + 3t\,\mathbf{j} + 2\sin t\,\mathbf{k}$. At the beginning of pipeline when $t = 0$, determine:

(i) unit tangent vector T.

(3 marks)

(ii) unit normal vector N.

(4 marks)

(iii) unit binomial vector B.

(4 marks)

(iv) curvature K.

(2 marks)

(b) The vector field of wind direction in Batu Pahat area is $F(x, y) = -xi + yj$. Sketch the 2D-graph manually for at least four points.

(4 marks)

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(c) The force field of pump power is $\mathbf{F}(x, y) = 2x\mathbf{i} + 3y\mathbf{j}$. By referring to **Figure Q4.1**, water particle moves from station 1 at point (-1, -1) to station 2 at point (3, -9).

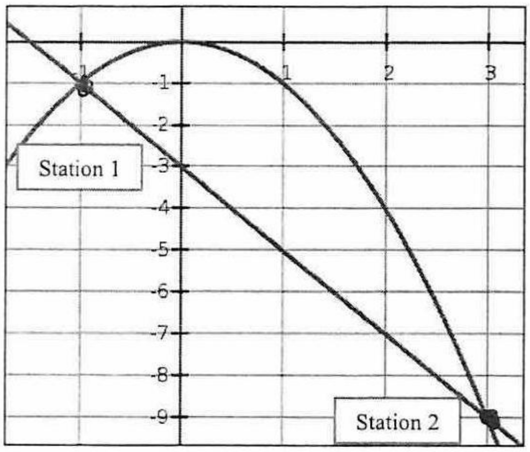

Figure Q4.1: Water particle movement from station 1 to station 2

(i) Determine the work done.

(6 marks)

(ii) Specify whether the force field is conservative or not. Justify your answer.

(2 marks)

- END OF QUESTIONS -

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APPENDIX A

Formula

Laplace Transforms

$\mathbf{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$			
$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s) - y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s) - sy(0) - \dot{y}(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$		

Partial Fraction

The denominator	**Partial Fraction**
$ax + b$	$\frac{A}{ax+b}$
$(ax+b)^2$	$\frac{A}{ax+b} + \frac{A_1}{ax+b^2}$
$ax^2 + bx + c$	$\frac{Ax+B}{ax^2+bx+c}$
$(ax^2 + bx + c)^2$	$\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2}$

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Tangent Plane: $z - z_0 = f_x(x_o, y_0)(x - x_0) + f_y(x_o, y_0)(y - y_0)$

Local Extreme Value: $G(x,y) = f_{xx}(x,y) \times f_{yy}(x,y) - [f_{xy}(x,y)]^2$

Case	$\boldsymbol{G(a,b)}$	**Result**
1	$G(a,b) > 0$ $f_{xx}(a,b) < 0$	$f(x,y)$ has a local maximum value at (a,b)
2	$G(a,b) > 0$ $f_{xx}(a,b) > 0$	$f(x,y)$ has a local minimum value at (a,b)
3	$G(a,b) < 0$	$f(x,y) has\ a\ saddle\ point\ at\ (a,b)$
4	$G(a,b) = 0$	inconclusive

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(\frac{y}{x})$ and

$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)r\,dz\,dr\,d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\theta$, $x^2 + y^2 + z^2 = \rho^2$, $0 \ll \theta \ll 2\pi$, $0 \ll \phi \ll \pi$ and $\iiint f(x,y,z)dV = \iiint f(\rho,\phi,\theta)\rho^2\sin\phi\, d\rho d\phi d\theta$

<u>For lamina</u>

Mass, $m = \iint_R \delta(x,y)\, dA$

Moment of mass: y-axis: $M_y = \iint_R x\delta(x,y)\, dA$ x-axis, $M_x = \iint_R y\delta(x,y)\, dA$

Center of mass, $(\bar{x}, \bar{y}) = (\frac{M_y}{m}, \frac{M_x}{m})$

Centroid for homogenous lamina: $\bar{x} = \frac{1}{area}\iint_R x\, dA$ $\bar{y} = \frac{1}{area}\iint_R y\, dA$

Moment inertia:

Y-axis: $I_y = \iint_R x^2\,\delta(x,y)\, dA$ x-axis: $I_x = \iint_R y^2\,\delta(x,y)\, dA$

Z-axis (or origin): $I_z = I_0 = \iint_R (x^2 + y^2)\,\delta(x,y)\, dA = I_x + I_y$

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For solid

Mass, $m = \iiint_G \delta(x,y)\, dV$

Moment of mass:

yz-plane: $M_{yz} = \iiint_G x\, \delta(x,y,z)\, dV$

xz-plane: $M_{xz} = \iiint_G y\, \delta(x,y,z)\, dV$

xy-plane: $M_{xy} = \iiint_G z\, \delta(x,y,z)\, dV$

Center of gravity, $(\bar{x}, \bar{y}, \bar{z}) = (\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m})$

Moment inertia:

$$I_y = \iiint_G (x^2 + z^2)\, \delta(x,y,z)\, dV$$

$$I_x = \iiint_G (y^2 + z^2)\, \delta(x,y,z)\, dV$$

$$I_z = \iiint_G (x^2 + y^2)\, \delta(x,y,z)\, dV$$

Directional derivative: $D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot u$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, then the divergence of $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The curl of

$$\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let C is a smooth curve given by $r(t) = x(t)i + y(t)j + z(t)k$, t is parameter, then

The unit tangent vector; $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The binormal vector: $B(t) = T(t) \times N(t)$

The curvature: $K = \frac{\mathbf{T}'(t)}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}' \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

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The radius of curvature: $\rho = 1/K$

Green Theorem: $\oint_C M(x,y)\,dx + N(x,y)\,dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dA$

Gauss Theorem: $\iint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_G \nabla.\mathbf{F}\,dV$

Stokes Theorem: $\oint_C \mathbf{F}.d\mathbf{r} = \iint_\sigma (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS$

Arc length, If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\|\,dt = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$

If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt$$

Trigonometric and Hyperbolic Identities

Trigonometric	
$\cos^2 x + \sin^2 x = 1$	$2\sin x\cos y = \sin(x+y) + \sin(x-y)$
$1 + \tan^2 x = \sec^2 x$	$2\sin x\sin y = -\cos(x+y) + \cos(x-y)$
$\cot^2 x + 1 = \mathrm{cosec}^2 x$	$2\cos x\cos y = \cos(x+y) + \cos(x-y)$
$\sin 2x = 2\sin x\cos x$	
$\cos 2x = \cos^2 x - \sin^2 x$	
$\cos 2x = 2\cos^2 x - 1$	
$\cos 2x = 1 - 2\sin^2 x$	
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	
$\sin(x\pm y) = \sin x\cos y \pm \cos x\sin y$	
$\cos(x\pm y) = \cos x\cos y \mp \sin x\sin y$	
$\tan(x\pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$	

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