UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME : DISCRETE MATHEMATICS

COURSE CODE : BWA 10603

PROGRAMME CODE : BWA

EXAMINATION DATE : JULY 2024

DURATION : 2 HOURS 30 MINUTES

INSTRUCTIONS :
1. ANSWER ALL QUESTIONS
2. THIS FINAL EXAMINATION IS CONDUCTED VIA
   ☐ Open book
   ☒ Closed book
3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

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Q1 Calculate:

  (a) −3 (mod 8),

  (b) 25 (mod 7),

  (c) 25 (mod 5),

  (d) −35 (mod 11).

(4 marks)

Q2 Let a function $f(n)$ maps from $V = \{1, 2, 3, 4\}$ into V be defined by $f(n) = 6 - n$.

  (a) Write $f(n)$ as a set of ordered pairs.

(2 marks)

  (b) Determine whether $f(n)$ is invertible or not.

(4 marks)

Q3 Prove the following argument is valid:

$$p \to \sim q$$
$$r \to q$$
$$\underline{\qquad r \qquad}$$
$$\sim p$$

(5 marks)

Q4 Negate the following statements:

  (a) All students live in the dormitories.

  (b) All mathematics majors are males.

  (c) Some students are 25 years old or older.

(3 marks)

Q5 Let $A = \{1, 2, 3, 4, 5\}$. Determine the truth value of the following statements:

  (a) $(\exists x \epsilon A)(x + 3 = 10)$;

  (b) $(\forall x \epsilon A)(x + 3 < 10)$.

(2 marks)

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Q6   Determine whether the following are true or false.

    (a) $793 \equiv 682 (\text{mod } 9)$.

    (b) $269 \equiv 413 (\text{mod } 12)$.

                                (4 marks)

Q7   Find the smallest integer in absolute value which is congruent modulo $m = 7$ to the following numbers:

    (a)   386,

    (b) -466.

    The integer should be in the set $\{-3, -2, -1, 0, 1, 2, 3\}$.

                                  (4 marks)

Q8   Exhibit the multiplication table for Z_7.

                                  (2 marks)

Q9   A history class contains **EIGHT (8)** male students and **SIX (6)** female students. Find the number of ways that the class can elect **ONE (1)** president and **ONE (1)** vice president.

                                  (2 marks)

Q10   Find the number of ways that **SEVEN (7)** people can arrange themselves around a circular table.

                                  (2 marks)

Q11   Find the number of distinct permutations that can be formed from all the letters of the word *SOCIOLOGICAL*.

                                  (2 marks)

Q12   Find n if $P(n, 2) = 72$.

                                  (4 marks)

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Q13 Let R be the following equivalence relation on the set $A = \{1, 2, 3, 4, 5, 6\}$:

$$R = \{(1,1), (1,5), (2,2), (2,3), (2,6), (3,2), (3,3),$$
$$(3,6), (4,4), (5,1), (5,5), (6,2), (6,3), (6,6)\}.$$

Find the equivalence classes of R.

(5 marks)

Q14 Consider the set $\mathbf{Z}$ of integers. Define aRb by $b = a^r$ for some positive integers r. show that R is reflexive, antisymmetric and transitive.

(5 marks)

Q15 For the given recurrence relation and initial conditions,

$$a_n = 3a_{n-1} + 10a_{n-2}; \quad a_0 = 5 \text{ and } a_1 = 11.$$

(a) Calculate a_2.

(2 marks)

(b) Derive the general solution.

(3 marks)

(c) Formulate the unique solution with the given initial conditions.

(5 marks)

Q16 Let a and b be real numbers such that $a < b$. Prove that there exist a real number, c such that $a < c < b$.

(10 marks)

Q17 Let $f(n) = 1 + 2 + \cdots + n, \ n \geq 1$. Show that $f(n) = \Theta(n^2)$.

(10 marks)

- **END OF QUESTIONS** -

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