Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	TECHNICAL MATHEMATICS II / CALCULUS
COURSE CODE	:	DAS 11103 / DAS 20803
PROGRAMME CODE	:	DAK / DAU
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Evaluate the limit for each of the following.

(i) $\lim_{x\to 2}\left(3+\sqrt{5x+6}\right)$.

(3 marks)

(ii) $\lim_{x\to 3}\dfrac{x^2-x-6}{x-3}$.

(4 marks)

(iii) $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x+3}-2}$.

(5 marks)

(iv) $\lim_{x\to\infty}\dfrac{5x^2+8x+3}{10x^2+7}$.

(4 marks)

(b) Given function $f(x)$ such that

$$f(x)=\begin{cases}6-mx, & x\le 2,\\ x^2-10, & 2<x\le 4,\\ \dfrac{12}{x-2}, & x>4.\end{cases}$$

where m is a constant.

(i) Find m if $\lim_{x\to 2} f(x)$ exists.

(4 marks)

(ii) Determine whether $f(x)$ is continuous at $x=4$ or not.

(5 marks)

Q2 (a) Calculate $\dfrac{dy}{dx}$ for the following functions.

(i) $y=(2x^5+3x-1)^3$.

(4 marks)

(ii) $y=(4+x^2)\sin 2x$.

(4 marks)

(iii) $y=\dfrac{2\ln x}{\sqrt{1-x}}$.

(4 marks)

(b) Using implicit differentiation, obtain $\frac{dy}{dx}$ for

$$x^4 + 3e^x y - 2\ln y = 2x^2.$$

(5 marks)

(c) Given $f(x) = 3x^{\frac{2}{3}}(x-5)$.

(i) Show that

$$f'(x) = \frac{5x-10}{x^{\frac{1}{3}}}.$$

(3 marks)

(ii) Hence, use part **Q2(e)(i)** to find all critical points of $f(x)$.

(5 marks)

Q3 (a) Solve the following integral:

(i) $$\int_2^3 \left(3s^2 + 3\sqrt[3]{s} - \frac{2}{s^5}\right) ds.$$

(5 marks)

(ii) $$\int \frac{\cos(\ln 2x) + x^2 \sin(3x)}{x} dx.$$

(8 marks)

(b) Evaluate the following by Simpsons Rule with $h = 0.2$:

$$\int_0^1 \sqrt{x^2+1}\, dx.$$

(7 marks)

(c) Evaluate the integral below using Trapezoidal Rule with $n = 4$. Give the answer to three decimal places.

$$\int_2^4 \frac{x^2}{\sqrt{x-1}}\, dx.$$

(5 marks)

Q4 (a) Consider the shaded region R enclosed by the curve $y = 4 - x^2$ and the line $y = 2 - x$ as shown in **Figure Q4.1** below.

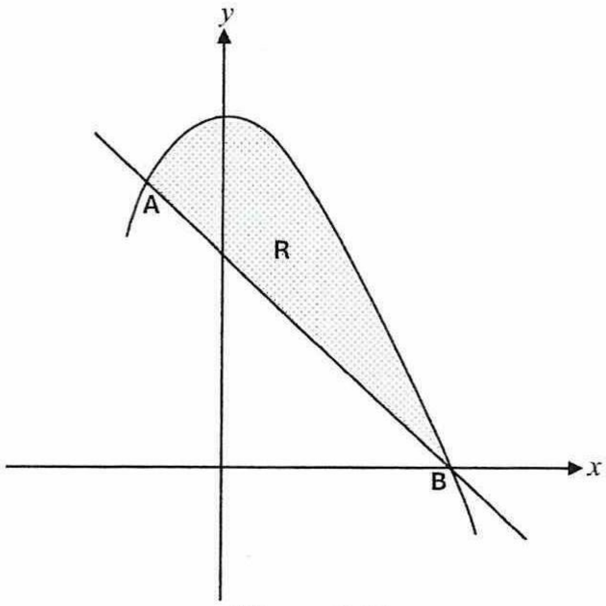

Figure Q4.1

(i) Find points A and B.

(4 marks)

(ii) Determine the area of the shaded region R.

(4 marks)

(b) **Figure Q4.2** depicts the region W bounded by the curve $x = 2(y - y^3)$ and line $x = 0$.

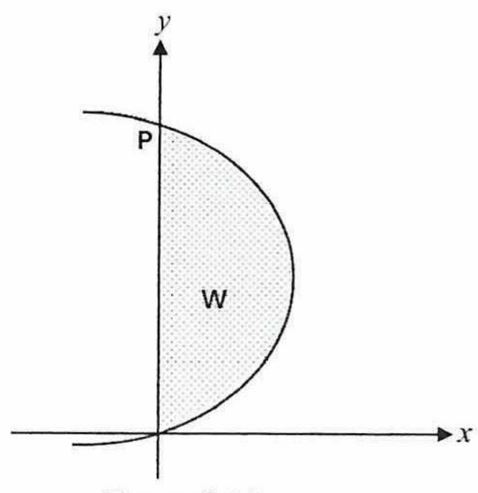

Figure Q4.2

(i) Identify the coordinates of point P.

(4 marks)

(ii) Use the cylindrical shells method to evaluate the volume of the solid in **Figure Q4.2** generated by rotating 360° the region W about $y = 0$.

(4 marks)

(c) Calculate the length of the curve $y = 6x^{\frac{3}{2}}$ from $0 \leq x \leq 2$.

(9 marks)

– END OF QUESTIONS –

FORMULAE

Table 1: Differentiation

$\frac{d}{dx}[k] = 0, \quad k \text{ is a constant}$	$\frac{d}{dx}[\sec x] = \sec x \tan x$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\frac{d}{dx}[\operatorname{cosec} x] = -\operatorname{cosec} x \cot x$
$\frac{d}{dx}[e^x] = e^x$	$\frac{d}{dx}[\cot x] = -\operatorname{cosec}^2 x$
$\frac{d}{dx}[\ln\|x\|] = \frac{1}{x}$	$\frac{d}{dx}[f(u(x))] = \frac{df}{du}\frac{du}{dx}$
$\frac{d}{dx}[\cos x] = -\sin x$	$\frac{d}{dx}[u \pm v] = \frac{du}{dx} \pm \frac{dv}{dx}$
$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Table 2: Integration

$\int k\,dx = kx + C, \quad k \text{ is a constant}$	$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$	$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$
$\int \frac{1}{x}\,dx = \ln\|x\| + C$	$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$
$\int \frac{1}{a+bx}\,dx = \frac{1}{b}\ln\|a+bx\| + C$	$\int u\,dv = uv - \int v\,du$

Area of a Region

$$A = \int_a^b [f(x) - g(x)]\,dx \quad \text{or} \quad A = \int_c^d [w(y) - v(y)]\,dy$$

Volume Cylindrical Shells

$$V = \int_a^b 2\pi x f(x)\,dx \quad \text{or} \quad V = \int_c^d 2\pi y f(y)\,dy$$

Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad \text{or} \quad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[\big(f(a) + f(b)\big) + 4\sum_{\substack{i=1 \\ i\text{ odd}}}^{n-1} f(a+ih) + 2\sum_{\substack{i=2 \\ i\text{ even}}}^{n-2} f(a+ih)\right]; \quad n = \frac{b-a}{h}$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(a+ih)\right]; \quad n = \frac{b-a}{h}$$