CONFIDENTIAL

Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	ENGINEERING MATHEMATIC
COURSE CODE	:	DAE 12003
PROGRAMME CODE	:	DAE
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) Solve the following limits.

(i) $\lim_{x \to 2} \frac{x^2 - 5x + 6}{2x^2 - 3x - 2}$.

(3 marks)

(ii) $\lim_{x \to -4} \frac{x + 4}{\sqrt{x+5} - 1}$.

(4 marks)

(b) Determine whether the following function is continuous at $x = 1$:

$$f(x) = \begin{cases} 1 + e^x & ; \quad x < 1 \\ \sqrt{3x^2 + 5} & ; \quad x = 1 \\ 2 + e^x - x & ; \quad x > 1 \end{cases}.$$

(6 marks)

(c) Calculate the values of a and b, so that functions $g(x)$ and $h(x)$ are continuous at $x = 2$:

$$g(x) = \begin{cases} x^2 + ax + 8 & ; \quad x \geq 2 \\ x - 2 & ; \quad x < 2 \end{cases} \text{ and } h(x) = \begin{cases} 4 - 2x & ; \quad x \geq 2 \\ b - x & ; \quad x < 2 \end{cases}.$$

(7 marks)

Q2 (a) Given $y = 3x^2 + \frac{4}{x^3}$. Show that, $x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 6y = 0$.

(5 marks)

(b) If $e^{xy^2} - 5x^2y^3 = 4x^3 - \sin 3y$, determine $\frac{dy}{dx}$ by using implicit differentiation.

(6 marks)

(c) Given parametric equations $x = 2t^3$ and $y = \frac{\sqrt{2 + t^2}}{t}$. Find $\frac{dy}{dx}$ for $t = 2$.

(6 marks)

(d) By using L'hopital's rule, evaluate $\lim_{x \to 0} \frac{1 - \cos x}{\sin 2x - x^2 - 2x}$.

(3 marks)

Q3 (a) Evaluate $\int x^2 e^{-1.5x} dx$ by using integration by parts.

(8 marks)

(b) **Figure APPENDIX A.1** shows a region ***R*** that is bounded by functions $y_1 = x^2 + 2$ and $y_2 = -x + 4$.

(i) Find the coordinates of the intersection points of the two curves.

(3 marks)

(ii) Calculate the area of the region ***R***.

(5 marks)

(c) **Figure APPENDIX A.2** shows a region ***R*** that is bounded by functions $y = 3 - x$, $y = 0$, $x = 0$ and $x = 2$. Find the volume of the solid when region ***R*** is revolved 360° about the y - axis.

(4 marks)

Q4 (a) Find the Laplace transform for following functions:

(i) $f(t) = 5e^{-\frac{1}{2}t} - \cosh\frac{3}{5}t$.

(2 marks)

(ii) $f(t) = t\left(\frac{\pi}{3} - t\right)^2 - \frac{5}{12}\sinh 4t$.

(4 marks)

(b) Use Multiplication by t^n to determine Laplace Transform for the following expressions:

$$f(t) = t\cos 2t.$$

(5 marks)

(c) Determine the inverse Laplace transform for the function below using partial fraction:

$$F(s) = \frac{3s^2 + 2s + 1}{(s^2+1)(s+2)(s+1)}.$$

(9 marks)

Q5 Solve the following differential equations using Laplace transform:

(a) $y'' + 7y' + 12y = e^{t}$; $y(0) = 0$ and $y'(0) = 1$.

(10 marks)

(b) $y' + 7y = \cos t$; $y(0) = 4$.

(10 marks)

- END OF QUESTIONS -

APPENDIX A

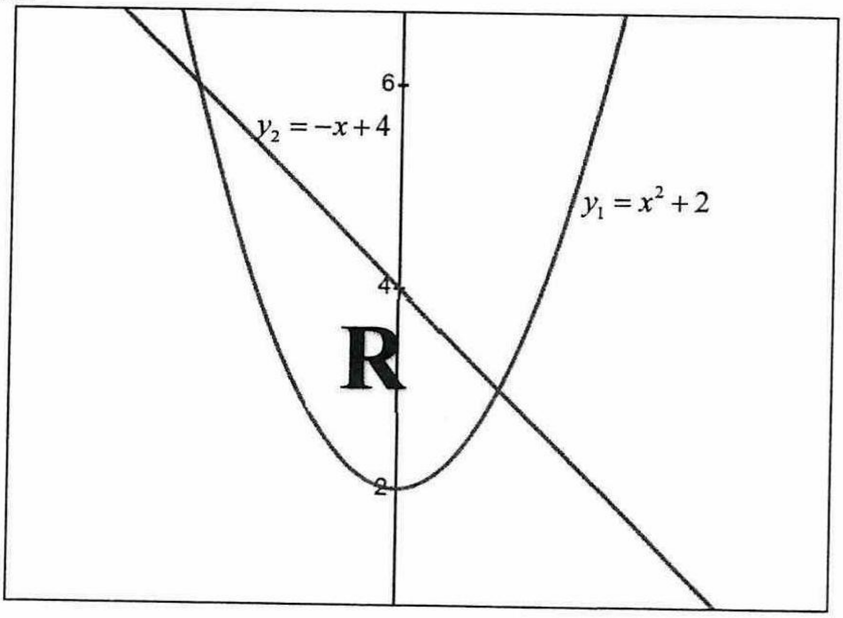

Figure APPENDIX A.1

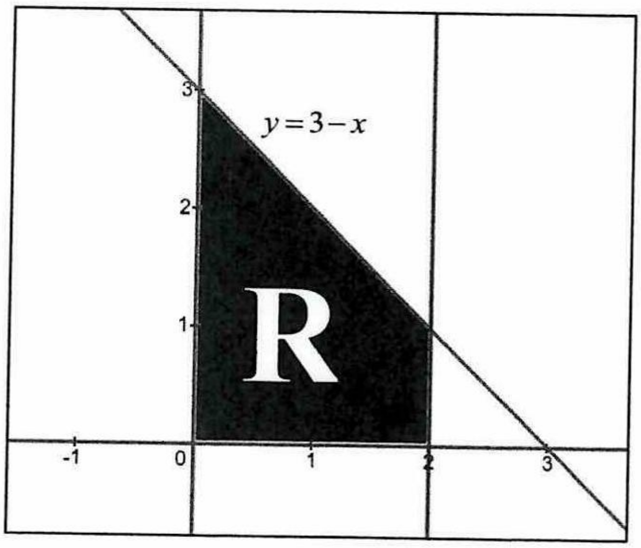

Figure APPENDIX A.2

LIST OF FORMULA

Table 1: Integration and Differentiation

Integration	Differentiation
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\frac{d}{dx} C = 0$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\frac{d}{dx} x^n = nx^{n-1}$
$\int \frac{1}{a-bx} dx = -\frac{1}{b} \ln\lvert a-bx\rvert + C$	$\frac{d}{dx} u^n = nu^{n-1} \cdot \frac{du}{dx}$, where $u = f(x)$
$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$	$\frac{d}{dx} \ln(u) = \frac{1}{u} \frac{du}{dx}$, where $u = f(x)$
$\int \sin(ax+b)\, dx = -\frac{1}{a} \cos(ax+b) + C$	$\frac{d}{dx} e^u = e^u \frac{du}{dx}$, where $u = f(x)$
$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$	$\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$, where $u = f(x)$
$\int \sec^2(ax+b) dx = \frac{\tan(ax+b)}{a} + C$	$\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$, where $u = f(x)$
$\int \text{cosec}^2(ax+b) dx = -\frac{\cot(ax+b)}{a} + C$	$\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$, where $u = f(x)$
$\int u\, dv = uv - \int v du$	$\frac{d}{dx} \cot u = -\csc^2 u \frac{du}{dx}$, where $u = f(x)$
$\int_a^b f(x) dx = F(b) - F(a)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
Area of Region $A = \int_a^b [f(x) - g(x)] dx$ **or** $A = \int_c^d [w(y) - v(y)] dy$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
Volume of Solid Region $V = 2\pi \int_a^b x[f(x) - g(x)] dx$ **or** $V = 2\pi \int_c^d y[w(y) - v(y)] dy$	Parametric Differentiation: $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

Table 2: Partial Fraction

$\frac{a}{(s+b)(s-c)} = \frac{A}{s+b} + \frac{B}{s-c}$
$\frac{a}{s(s-b)(s-c)} = \frac{A}{s} + \frac{B}{s-b} + \frac{C}{s-c}$
$\frac{a}{(s+b)^2} = \frac{A}{s+b} + \frac{B}{(s+b)^2}$
$\frac{a}{(s+b)(s^2+c)} = \frac{A}{(s+b)} + \frac{Bs+C}{(s^2+c)}$

Table 3: Laplace and Inverse Laplace Transforms

$L\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$	
$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n, n=1,2,\ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
First Shift Theorem	
$e^{at}f(t)$	$F(s-a)$
Multiply with t^n	
$t^n f(t), n=1,2,\ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$
Initial Value Problem	
$L\{y(t)\}=Y(s)$	
$L\{y'(t)\}=sY(s)-y(0)$	
$L\{y''(t)\}=s^2Y(s)-sy(0)-y'(0)$	

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