Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	ALGEBRA
COURSE CODE	:	DAS 10103
PROGRAMME CODE	:	DAU
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Find the pattern of the following sequence 1,6,11…

(3 marks)

(b) Solve the equation $\log_x 135 = \log_x 5 + 3$

(6 marks)

(c) Use De Moivre's Theorem to find all the square roots of $z = 2 - i\sqrt{3}$.

(7 marks)

(d) Write $\frac{4}{1+\sqrt{5}}$ in simplest form

(4 marks)

Q2 (a) *ABCD* is a parallelogram such that $\overrightarrow{\mathbf{AC}} = 3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$. If the position vector of **A** and **B** are $3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$ and $5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ respectively. Find the position vector of point **D**.

(5 marks)

(b) Find vector equation of plane containing **P**(4,-1,2), **Q**(2,0,3) and **R**(-1,0,2)

(9 marks)

(c) A straight line passes through point $\mathbf{K} = (-1, -4, 5)$ and $\mathbf{L} = (-2, 6, 1)$.

(i) Find a vector equation of the straight line.

(2 marks)

(ii) State the parametric and symmetric equations of the straight line.

(4 marks)

Q3 (a) Solve the following exponential equation: $\left(\frac{1}{3}\right)^{-2-2x} = 729^{-1}$

(5 marks)

(b) prove $cos(\pi - x) = -cos\, x$.

(3 marks)

(c) By using double angle formulae, evaluate the value of $\cos 4(90^\circ)$.

(4 marks)

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(d) Given $2\sin\theta + 3\cos\theta = r\sin(\theta+\alpha)$ and $0 \le \theta \le 2\pi$.

(i) Determine r and α.

(4 marks)

(ii) Thus, estimate the value of θ if $2\sin\theta + 3\cos\theta = 3$.

(4 marks)

Q4 (a) Find the value of x that satisfies the equation

$$ln(4x+1) - ln(3x-2) = ln\,5 + ln\,x.$$

(4 marks)

(b) Solve $7\,log_2\,x - 6\,log_x\,2 = 19$

(6 marks)

(c) Calculate the root of the $f(x) = x^4 + 2x^2 - x - 3$ in the interval $[0, 1.5]$ using Secant method. Iterate until $|f(x_i)| < \varepsilon = 0.005$. Show your calculation in three decimal places.

(10 marks)

Q5 (a) Solve the following system of linear equations by using inverse matrix:

$$\begin{aligned} x + 2y - 4z &= -4 \\ 2x + 5y - 9z &= -10. \\ 3x - 2y + 3z &= 11 \end{aligned}$$

(11 marks)

(b) Let $A = \begin{pmatrix} 4 & 0 & 3 \\ 2 & -3 & 1 \\ 5 & 4 & -5 \end{pmatrix}$, $B = \begin{pmatrix} 6 & 0 & 5 \\ 5 & 1 & 3 \\ 2 & -2 & 9 \end{pmatrix}$

Find value of

(i) A-$2B$

(3 marks)

(ii) AB

(4 marks)

(iii) $(AB)^T$

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(2 marks)

- END OF QUESTIONS -

LIST OF FORMULA

Polynomials

$$\log_a x = \frac{\log_a x}{\log_a b} \qquad a^3 - b^3 = (a-b)(a^2+ab+b^2) \qquad a^3 + b^3 = (a+b)(a^2-ab+b^2)$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}, \qquad x^2+bx+c = \left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c, \qquad x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

Sequence and Series

$$\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$x_n = a+(n-1)d \qquad S_n = \frac{n}{2}[2a+(n-1)d], \qquad S_n = \frac{n}{2}(a+u_n)$$

$$x_n = ar^{n-1}, \quad S_n = \frac{a(r^n-1)}{r-1}, r>1 \ \textbf{OR}\ S_n = \frac{a(1-r^n)}{1-r}, r<1, \quad S_\infty = \frac{a}{1-r}.$$

$$x_n = S_n - S_{n-1}$$

$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots\ldots$$

Trigonometry

$$\sin^2 x + \cos^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

LIST OF FORMULA

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}, \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}, \quad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$a\sin\theta + b\cos\theta = r\sin(\theta+\alpha) = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = (r\cos\alpha)\sin\theta + (r\sin\alpha)\cos\theta$ and

$a = r\cos\alpha$ and $b = r\sin\alpha$

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}}{a_{11}}, \quad x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}, \quad x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$$

Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

LIST OF FORMULA

Table 1: Vector

$\lvert\mathbf{u}\rvert = \sqrt{a^2 + b^2 + c^2}$	$\hat{\mathbf{u}} = \dfrac{\mathbf{u}}{\lvert\mathbf{u}\rvert}$
$\mathbf{u} \bullet \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$	$\mathbf{u} \bullet \mathbf{v} = \lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert\cos\theta$
$\theta = \cos^{-1}\left(\dfrac{\mathbf{u}\bullet\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}\right)$	$A = \dfrac{1}{2}\lvert\mathbf{u}\times\mathbf{v}\rvert$
$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$	$x = x_0 + a_1t$ $y = y_0 + a_2t$ $z = z_0 + a_3t$
$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$	

Table 2: Complex Number

$z = a + bi$ $\bar{z} = a - bi$	$z = r(\cos\theta + i\sin\theta)$
$r = \sqrt{a^2 + b^2}$	$\theta = \tan^{-1}\left(\dfrac{b}{a}\right)$
$z_1z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$	
$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$	
$z = re^{i\theta}$	$z^n = r^ne^{in\theta}$
$z^{\frac{1}{n}} = r^{\frac{1}{n}}e^{\left(\frac{\theta+2k\pi}{n}\right)i}$	$z^n = r^n[\cos n\theta + i\sin n\theta]$
$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\dfrac{\theta + 2k\pi}{n} + i\sin\dfrac{\theta + 2k\pi}{n}\right)$	