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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAS 20603
PROGRAMME	:	2 DAE
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A) ANSWER **ALL** QUESTIONS SECTION B) ANSWER **THREE (3)** QUESTIONS ONLY

TERBUKA

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

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SECTION A

Q1 (a) The motion of a particle satisfies the equation

$$\frac{d^2y}{dt^2}+6\frac{dy}{dt}+9y=0$$

which satisfying the initial condition $y(0)=2$ and $y'(0)=1$. Find the particular solution for the second order homogeneous differential equation of $y(t)$.

(9 marks)

(b) By using method of undetermined coefficient, find the general solution by finding the homogeneous and particular solution of the following second order non-homogeneous differential equation below

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=3x^2e^x$$

(11 marks)

Q2 Use the method of variation of parameters to find the particular solution of the given non-homogeneous equation. Then find the general solution of the equation.

(a) $$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{3}{2e^{4x}}$$

(10 marks)

(b) $$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=2x^2-3x$$

(10 marks)

SECTION B

Q3 (a) Evaluate $\int 4\cos 3x\, e^{2x}\, dx$ by using tabular method.

(6 marks)

(b) By using substitution technique, evaluate

$$\int_0^3 \left[\frac{x^3}{(2x^2+1)^3}\right] dx.$$

(8 marks)

(c) Solve $\int_1^3 \frac{x+x^3}{\sqrt{3e^{2x}}}\, dx$ by using Simpson's rule, using $h = 0.25$. Write the answer to 2 decimal places.

(6 marks)

Q4 (a) From **Figure Q4 (a)**, find the area of the region enclosed by the curve

$$y = 2 + x - x^2 \text{ and } y = -x - 1$$

(7 marks)

(b) Use cylindrical shells to find the volume of the solid that results when the region enclosed by $y = 2x$ and $y^2 = 4x$ is revolved about the $y-$axis.
Refer **Figure Q4 (b)**.

(6 marks)

(c) Find the arc length of the curve $y = \frac{1}{3}\left(x^2 + 2\right)^{3/2}$ from $x = -1$ to $x = 1$.

(7 marks)

Q5 (a) Given the first order separable differential equation

$$2x\frac{dy}{dx} = a + by + y^2$$

(i) Solve the differential equation if $a = 1$ and $b = -2$.

(4 marks)

(ii) Solve the differential equation if $a = -2$ and $b = -1$.

(5 marks)

(b) Given the first order linear differential equation

$$(x^2 - 1)\frac{dy}{dx} + 8y = 2\left(\frac{x+1}{x-1}\right)^4$$

(i) If the integrating factor is $\left(\frac{x-1}{x+1}\right)^4$, find the value of k.

(5 marks)

(ii) Then solve the differential equation with the condition $y(2) = 3$.

(6 marks)

Q6 In a murder investigation a corpse was found by Inspector Ali at exactly 8:00 pm. Being alert, he measures the temperature of the body and finds it to be $70°F$. Two hours later, Inspector Ali again measure the temperature of the corpse and finds it to be $60°F$. Assuming that the victim's body temperature was normal ($98.6°F$) prior to death.

(a) By following the Newton's Law of Cooling,

$$\frac{dT}{dt} = -k(T - T_S)$$

where T is temperature of the body, T_S is temperature surrounding body and k is the constant proportionality. Show that cooling equation can be written as $T = (T_0 - T_S)e^{-kt} + T_S$ if the initial condition $T = T_0$.

(9 marks)

(b) If the room temperature is $50°F$, when did the murder occur?

(11 marks)

Q7 (a) Given the differential equation of growth of decay problem can be formulated as

$$\frac{dN}{dt} = kN$$

where $N(t)$ is the amount of material present and k is a constant proportionality. Using separable method, solve for $N(t)$.

(4 marks)

(b) Carbon-14 has a half life of 5730 years.

(i) If the initial amount of carbon-14 is 3000 gram, find how much is left after 2000 years.

(7 marks)

(ii) Determine when it will be if 350 grams carbon-14 left.

(3 marks)

(iii) Find how much it is left after 2000 years if the half life duration change to 3000 years.

(6 marks)

- END OF QUESTION –

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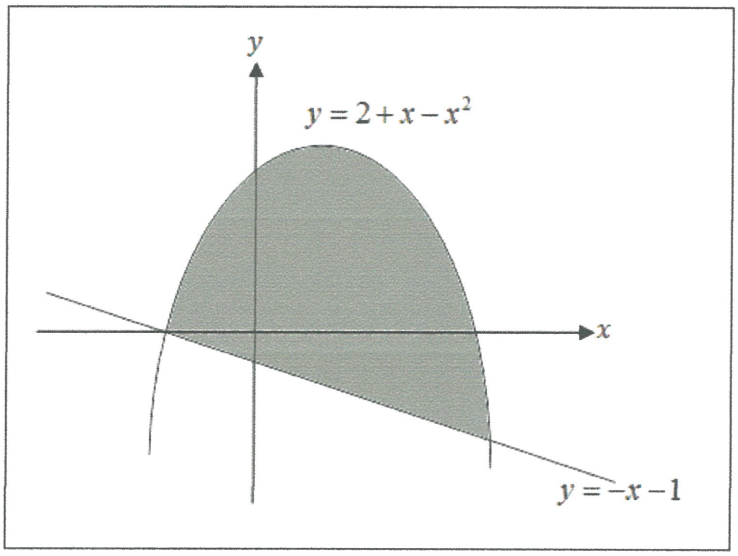

Figure Q4 (a)

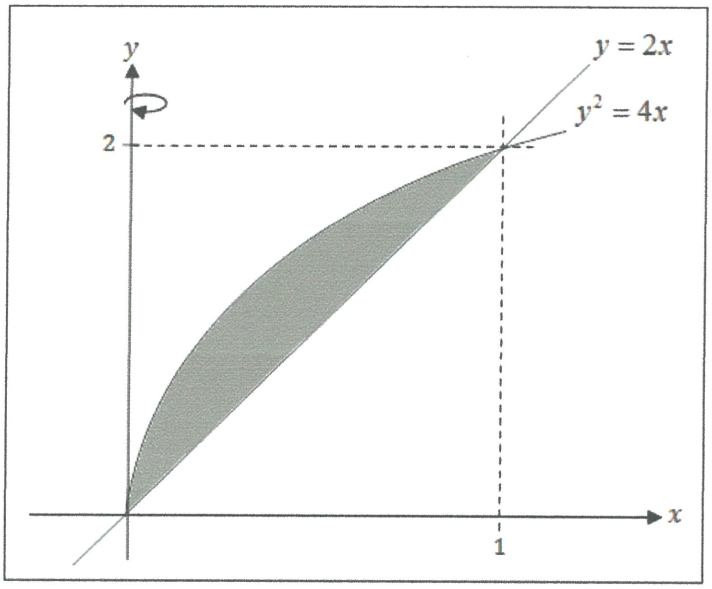

Figure Q4 (b)

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Formulae

Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$;
Characteristic equation : $am^2 + bm + c = 0$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos \beta x + B\sin \beta x)$

Method of Variation of Parameters

Homogeneous solution, $y_h(x) = Ay_1 + By_2$

Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

$u_1 = -\int \frac{y_2 f(x)}{aW} dx$ $u_2 = \int \frac{y_1 f(x)}{aW} dx$

Particular solution, $y_p = u_1 y_1 + u_2 y_2$

Final solution, $y(x) = y_h(x) + y_p(x)$

Method Of Undetermined Coefficients

Case	$F(x)$	$y_p(x)$
1	Simple polynomial: $A_0 + A_1 x + \ldots + A_n x^n$	$x^r(B_0 + B_1 x + \ldots + B_n x^n)$, $r = 0, 1, 2,$
2	Exponential function: $Ce^{\alpha x}$	$x^r(ke^{\alpha x})$, $r = 0, 1, 2,$
3	Simple trigonometry: $C\cos \beta x$ or $C\sin \beta x$	$x^r(p\cos \beta x + q\sin \beta x)$, $r = 0, 1, 2,$

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Trigonometry

$$\cos^2 x + \sin^2 x = 1$$

$$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$2\sin x \cos y = \sin(x+y) + \sin(x-y)$$
$$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$$
$$2\cos x \cos y = \cos(x+y) + \cos(x-y)$$

Differentiation and Integration

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}\ln\lvert ax+b\rvert = \frac{1}{ax+b}$	$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\sin ax = a\cos ax$	$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$
$\frac{d}{dx}\cos ax = -a\sin ax$	$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$

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Definite Integration

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Length of Curve

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$
$$= \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

Area

$$A = \int_a^b \left[f(x) - g(x)\right]dx$$
$$= \int_c^d \left[f(y) - g(y)\right]dy$$

Volume Cylindrical Method

$$V = 2\pi\int_a^b x\,f(x)\,dx$$
$$= 2\pi\int_c^d y\,f(y)\,dy$$

Area of Surface Revolution

$$S = 2\pi\int_a^b y(x)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$
$$= 2\pi\int_c^d x(y)\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$