Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAS 20403
PROGRAMME	:	2 DAA / 2 DAM / 3 DAA / 3 DAM
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	A. ANSWER **ALL** QUESTIONS IN **SECTION A** B. ANSWER **THREE (3)** QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

SECTION A

Q1 a) Find the inverse of the following Laplace transform.

(i) $\frac{3}{s}+\frac{12}{s^4}-\frac{4}{s+3}$

(3 marks)

(ii) $\frac{10}{s^2+25}-\frac{8s}{4s^2+16}$

(4 marks)

(iii) $\frac{2s-5}{(s-3)^2}$ [Hint : write 2s – 5 as 2s – 6 +1]

(4 marks)

(b) (i) Express $\frac{5s+1}{s^2-s-12}$ into partial fraction form.

(5 marks)

(ii) Find the inverse Laplace of **Q(b)(i)**.

(4 marks)

Q2 Solve the differential equation below by using Laplace transform.

(a) $y''+2y'-3y=8e^{-2t}, \quad y(0)=0, \quad y'(0)=1$

(10 marks)

(b) $y''-y=\sin t, \quad y(0)=0, \quad y'(0)=1$

(10 marks)

SECTION B

Q3 (a) Given

$$\left(2y+x^2+1\right)+\left(2xy-9x^2\right)\frac{dx}{dy}=0$$

(i) Show that the equation is an exact ordinary differential equation.

(4 marks)

(ii) Find the general solution of the equation.

(7 marks)

(b) Find the solution of the given IVP differential equation.

$$x\frac{dy}{dx}+2y=x^2-x+1 \quad , \quad y(1)=\frac{1}{2}$$

(9 marks)

Q4 (a) According to Newton's law of cooling, the rate at which a body cools is given by the equation

$$\frac{dT}{dt}=-k(T-T_e)$$

where T is the temperature in degrees Celsius at time t, k is a proportionality constant and T_e the environment temperature.

(i) By integrating the separable equation, find T.

(3 marks)

(ii) If a body cools from 100 °C to 60 °C in 10 minutes, and the environment temperature keeps constant at 20 °C, find the time needed for the body to cool down to 30 °C.

(8 marks)

(b) The population growth rate law is given by the following equation

$$\frac{dP}{dt}=kP$$

where P is the present population and k is a proportional constant.

(i) If at $t = 0$ and $P = P_o$, find P at time t.
[Hint: Integrating the separable equation to find P]

(4 marks)

(ii) Use equation P from **Q4(b)(i),** to find the number of bacteria present in a food sample after 10 hours if after 4 hours the number increasing to 5.0×10^5 from initial number at 10,000 of bacteria.

(5 marks)

Q5 (a) Using undetermined coefficient method, solve the equation

$$y''+3y'-4y=\cos x, \quad y(0)=1, \quad ,y'(0)=0$$

(10marks)

(b) Using variation parameter method, solve the equation

$$y''+3y'+2y=4e^{x}$$

(10 marks)

Q6 (a) Find the Laplace transform of the following function

(i) $\mathcal{L}\{3+12t^2\}$

(3 marks)

(ii) $\mathcal{L}\{(t+1)^3\}$

(3 marks)

(iii) $\mathcal{L}\{\sinh 7t - e^{-\frac{1}{2}t}\}$

(3 marks)

(iv) $\mathcal{L}\{e^{2t}(t-2)(t-3)\}$

(4 marks)

(b) Show that

$$\mathcal{L}\{t^2 \sin 2t\} = \frac{4(3s^2-4)}{(s^2+4)^3}$$

(7 marks)

Q7 (a) Solve the second order homogenous differential equation.

$$y''+2y'+y=0, \quad y(0)=1, \quad y'(0)=2$$

(7 marks)

(b) Find the Inverse Laplace Transform of $\dfrac{3}{s\left(s^2-1\right)}$ by using partial fraction.

(6 marks)

(c) Use Laplace Transform to solve the differential equation $y'+2y=e^{5t}$, given $y(0)=0$

(7 marks)

-END OF QUESTIONS –

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Formulae

Table 1 : Laplace Transformation

$f(t)$	$F(s)$	$f(t)$	$F(s)$
k	$\frac{k}{s}$	$\sinh at$	$\frac{a}{s^2-a^2}$
$t^n,\ n=1,2,..$	$\frac{n!}{s^{n+1}}$	$\cosh at$	$\frac{s}{s^2-a^2}$
e^{at}	$\frac{1}{s-a}$	$e^{at}f(t)$	$F(s-a)$
$\sin at$	$\frac{a}{s^2+a^2}$	$t^n f(t),\ n=1,2,..$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$\cos at$	$\frac{s}{s^2+a^2}$		

$$L\{y(t)\} = Y(s)$$

$$L\{y'(t)\} = sY(s) - y(0)$$

$$L\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$$

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Table 2 : Integration and Differentiation

Integration	Differentiation
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\frac{d}{ds}(uv) = v\frac{du}{ds} + u\frac{dv}{ds}$
$\int \frac{1}{x} dx = \ln\|x\| + C$	$\frac{d}{ds}\left(\frac{u}{v}\right) = \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$
$\int \frac{1}{a-bx} dx = -\frac{1}{b}\ln\|a-bx\| + C$	$\frac{d}{ds}\left(e^{ax}\right) = ae^{ax}$
$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$	$\frac{d}{ds}(\sin ax) = a\cos ax$
$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$	$\frac{d}{ds}(\cos ax) = -a\sin ax$
$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$	$\frac{d}{ds}\left(x^n\right) = nx^{n-1}$
$\int udv\ dx = uv - \int vdu$	$\frac{d}{ds}(uv) = v\frac{du}{ds} + u\frac{dv}{ds}$

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Table 3 : Characteristic Equation and General Solution

Homogeneous Differential equation : $ay'' + by' + cy = 0$

Characteristic equation : $am^2 + bm + c = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 4 : Particular Solution of Nonhomogeneous Equation

$$ay'' + by' + cy = f(x)$$

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1} x^{n-1} + \ldots + A_1 x + A_0$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r (Pe^{\alpha x})$
$C\cos\beta x$ atau $C\sin\beta x$	$x^r (p \text{ kos } \beta x + q \sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0) e^{\alpha x}$
$P_n(x)\begin{cases} kos\beta x & atau \\ \sin\beta x \end{cases}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0)\, kos\, \beta x$ + $x^r (B_n x^n + B_{n-1} x^{n-1} + \ldots + B_1 x + B_0) \sin\beta x$

Notes : r is the smallest non negative integers to ensure no alike terms between $y_p(x)$ and $y_h(x)$.

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Table 5 : Variation of Parameters Method.

Homogeneous solution, $y_h(x) = Ay_1 + By_2$
Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$
$u_1 = -\int \frac{y_2 f(x)}{aW} dx + A$ $\qquad$ $u_2 = \int \frac{y_1 f(x)}{aW} dx + B$
General solution, $y(x) = u_1 y_1 + u_2 y_2$

Table 6 : Trigonometry Identities

$\sin^2 t + \cos^2 t = 1$
$\sin^2 t = \frac{1}{2}(1 - \cos 2t)$
$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$

Table 7 : Partial Fraction

$\frac{a}{(s+b)(s-c)} = \frac{A}{s+b} + \frac{B}{s-c}$
$\frac{a}{s(s-b)(s-c)} = \frac{A}{s} + \frac{B}{s-b} + \frac{C}{s-c}$
$\frac{a}{(s+b)^2} = \frac{A}{s+b} + \frac{B}{(s+b)^2}$
$\frac{a}{(s+b)(s^2+c)} = \frac{A}{(s+b)} + \frac{Bs+C}{(s^2+c)}$

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