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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	ALGEBRA
COURSE CODE	:	DAS 10103
PROGRAMME CODE	:	1DAA / 1DAM / 1DAE / 1DAU/ 1DAT
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **SECTION A** B) ANSWER **THREE (3)** QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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SECTION A

Q1 (a) Let $\boldsymbol{u} = 4\boldsymbol{i} + \boldsymbol{k}$, $\boldsymbol{v} = 2\boldsymbol{i} - \boldsymbol{j}$ and $\boldsymbol{w} = a\boldsymbol{i} - \boldsymbol{j} - 4b\boldsymbol{k}$. Find:

(i) $4\boldsymbol{u} - 3\boldsymbol{v} + \boldsymbol{w}$,

(4 marks)

(ii) $\boldsymbol{u} \times \boldsymbol{v}$,

(3 marks)

(iii) The value of a and b if $4\boldsymbol{u} - 3\boldsymbol{v} + \boldsymbol{w} = \boldsymbol{u} \times \boldsymbol{v}$.

(3 marks)

(b) Find the parametric equation of the line that passes through points $S(2, -1, 3)$ and $T = (-1, 1, -2)$.

(4 marks)

(c) Given three points $P = (1, 2, -1)$, $Q = (2, 3, 1)$ and $R = (3, -1, 2)$. Find:

(i) the normal vector, $\mathbf{n}$ where $\mathbf{n} = \boldsymbol{PQ} \times \boldsymbol{PR}$,

(4 marks)

(ii) the equation of the plane with points P, Q and R on it.

(2 marks)

Q2 (a) If $v = 2 - 3i$ and $w = 3 + i$, express $z = \frac{2v+w}{w}$ in the form if $a + bi$.

(5 marks)

(b) Given $z_1 = 1 + i\sqrt{3}$ and $z_2 = 6 + 8i$.

(i) Express $\frac{z_1}{z_2}$ in polar form.

(6 marks)

(ii) Express $z_1 z_2$ in polar form.

(4 marks)

(c) Applying De Moivre's Theorem, calculate $\left(1 + i\sqrt{3}\right)^5$ in the form of $a + bi$.

(5 marks)

SECTION B

Q3 (a) (i) Find the value of x, $3^x = \frac{243^x}{729}$.

(3 marks)

(ii) Solve $(2^x)^2 - 80^x + 256 = 0$.

(6 marks)

(b) Solve the equation $5\log_x 2 - \log_2 \sqrt{x} = 4\frac{1}{2}$.

(8 marks)

(c) Simplify the expression below. Assume that *x*, *y* and *z* are positive.

$$\sqrt[4]{4x^3y}\,.\,\sqrt[4]{8x^2y^3z^5}$$

(3 marks)

Q4 (a) Find the root of the equation $2x - \sin x - \frac{1}{2} = 0$ in the interval $[1, 2]$ using Secant method. Iterate until $|f(x_i)| < \varepsilon = 0.005$. Show your calculation in three decimal places.

(7 marks)

(b) Express $\frac{4x-2}{(x+3)(x-2)^2}$ in the form of partial fraction.

(8 marks)

(c) Using Binomial expansion, find the first three terms of $\frac{1}{(4+x)^2}$.

(5 marks)

Q5 (a) (i) Find the pattern of the following sequence 1, 6, 11, . . .

(3 marks)

(ii) Find the sum of the sequence $\sum_{k=1}^{7}(2k^3 - 8k + 7)$.

(3 marks)

(b) Given that the n^{th} term of arithmetic sequence is $a_n = 23 + 2(n-1)$.

(i) Find the value of first term, *a* and its common difference, *d*.

(3 marks)

(ii) Find S_{10}.

(3 marks)

(c) A geometric sequence is defined as $30, 20, \frac{40}{3}, \frac{80}{9}, \ldots$

(i) Find the value of common ratio, *r*.

(2 marks)

(ii) Calculate the tenth term, a_{10}.

(2 marks)

(iii) State whether this series converges or diverges. State your reason.

(2 marks)

(iv) If it is converges, evaluate its summation, S_∞.

(2 marks)

Q6 (a) By using sum and difference identities, simplify

$$\frac{1+\tan x}{1-\tan x}.$$

(5 marks)

(b) Without using calculator, find the value of:

(i) $\cos 120°$ by using the double angle formula.

(4 marks)

(ii) $\sin 15°$ by using the half angle formula.

(4 marks)

(c) Given $6\cos\theta + 7\sin\theta = r\sin(\theta+\alpha)$ and $0 \le \theta \le 2\pi$.

(i) Find r and α.

(3 marks)

(ii) Thus, find the value of θ if $6\cos\theta + 7\sin\theta = 8$.

(4 marks)

Q7 (a) Given $A = 3\begin{pmatrix} 1 & 4 \\ 3 & 2 \\ -2 & 5 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 3 & 2 \\ 2 & 5 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} -5 & 3 & 9 \\ 2 & 7 & 2 \end{pmatrix}$. Solve the following epression:

(i) $\boldsymbol{AC} + \boldsymbol{AB}$.

(4 marks)

(ii) Show that $(\boldsymbol{B}+\boldsymbol{C})^T = \boldsymbol{B}^T + \boldsymbol{C}^T$.

(3 marks)

(b) Given

$$4x + 8y + 3z = 2$$
$$3x + 5y + z = -3$$
$$x + 4y + 3z = 5$$

(i) Write the matrix equation $\boldsymbol{AX} = \boldsymbol{B}$ of the system equation.

(1 mark)

(ii) Find the determinant of matrix $\boldsymbol{A}$.

(3 marks)

(iii) Solve the above system for x, y, and z by using the Gauss-Jordan elimination method. Do this following operation in order:

$R_3 \Leftrightarrow R_1$,

$R_2 - 3R_1$,

$R_3 - 4R_1$,

$-\frac{1}{7}R_2$,

$R_3 + 8R_2$,

$7R_3$,

$R_1 - 4R_2$,

$R_1 + \frac{11}{7}R_3$,

$R_2 - \frac{8}{7}R_3$.

(9 marks)

-END OF QUESTIONS –

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FORMULA

Polynomials

$$\log_a x = \frac{\log_a x}{\log_a b}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c, \qquad x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

Sequence and Series

$$\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$u_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d], \qquad S_n = \frac{n}{2}(a + u_n)$$

$$u_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r-1}, r > 1 \ \textbf{OR} \ S_n = \frac{a(1-r^n)}{1-r}, r < 1, \quad S_\infty = \frac{a}{1-r}.$$

$$u_n = S_n - S_{n-1}$$

$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots\ldots$$

Trigonometry

$$\sin^2 x + \cos^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

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$$\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}\,,\quad \cos\frac{\theta}{2}=\pm\sqrt{\frac{1+\cos\theta}{2}}\,,\quad \tan\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$a\sin\theta+b\cos\theta=r\sin(\theta+\alpha)=r(\sin\theta\cos\alpha+\cos\theta\sin\alpha)=(r\cos\alpha)\sin\theta+(r\sin\alpha)\cos\theta$ and

$a=r\cos\alpha$ and $b=r\sin\alpha$

$$x_1^{(k+1)}=\frac{b_1-a_{12}x_2^{(k)}-a_{13}x_3^{(k)}}{a_{11}}\,,\quad x_2^{(k+1)}=\frac{b_2-a_{21}x_1^{(k+1)}-a_{23}x_3^{(k)}}{a_{22}}\,,\quad x_3^{(k+1)}=\frac{b_3-a_{31}x_1^{(k+1)}-a_{32}x_2^{(k+1)}}{a_{33}}$$

Matrices

$$A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix},\ |A|=a_{11}\begin{vmatrix}a_{22}&a_{23}\\a_{32}&a_{33}\end{vmatrix}-a_{12}\begin{vmatrix}a_{21}&a_{23}\\a_{31}&a_{33}\end{vmatrix}+a_{13}\begin{vmatrix}a_{21}&a_{22}\\a_{31}&a_{32}\end{vmatrix}$$

Vector

$$\cos\theta=\frac{\boldsymbol{a}\bullet\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}\quad \textbf{OR}\quad \boldsymbol{a}\bullet\boldsymbol{b}=x_1x_2+y_1y_2+z_1z_2\,,\quad \boldsymbol{a}\times\boldsymbol{b}=\begin{vmatrix}\boldsymbol{i}&\boldsymbol{j}&\boldsymbol{k}\\x_1&y_1&z_1\\x_2&y_2&z_2\end{vmatrix}$$

$|a|=\sqrt{x^2+y^2+z^2}$,

$x=x_0+a_1t,\quad y=y_0+a_2t,\quad z=z_0+a_3t$ and $\dfrac{x-x_o}{a_1}=\dfrac{y-y_o}{a_2}=\dfrac{z-z_o}{a_3}$

$A(x-x_o)+B(y-y_o)+C(z-z_o)=0$

Complex number

TERBUKA

$r=\sqrt{x^2+y^2}\qquad \tan\theta=\dfrac{y}{x}$