UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2023/2024

COURSE NAME	:	CALCULUS
COURSE CODE	:	BFC 15003
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JANUARY / FEBRUARY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 Answer all the following questions.

(a) If $y = \dfrac{x^2}{x-2}$, find the $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

(10 marks)

(b) Compute the derivatives of $y = (2x+5)^{12}(x^3+5)^{13}$.

(5 marks)

(c) Determine the derivatives of $y = \ln(3x-1)$.

(4 marks)

(d) If $3y^2 - 2x^2 = 2xy$, determine the $\dfrac{dy}{dx}$.

(6 marks)

Q2 An experimental work on reinforced concrete beam was conducted to investigate the behaviour of the beam under flexure. From the experimental result of control beam testing, load versus deflection (P-δ) graph was plotted and can be written as a function of:

$$P = -0.791\delta^2 + 16.802\delta + 3.031$$

(a) Find the critical point.

(4 marks)

(b) Determine the intervals where the function is increasing, decreasing, concave up and concave down.

(9 marks)

(c) Explain what the critical number represents in this function.

(2 marks)

(d) Concrete cubes were casted to identify the compressive strength of the concrete. The cubes were heated in an oven for one hour. The temperature of the oven was set to be 80°C. Upon heated, each side of the cube increases at a rate of 0.005 m/s.

(i) Determine the rate its volume increasing when each side has length of 0.151 m. Give your answer in cm^3/s.

(5 marks)

(ii) Determine the rate its surface area increasing when each side has length of 0.15 m. Give your answer in m^2/s.

(5 marks)

Q3 Evaluate the following integrals using a suitable integration method:

(a) $\int_{-\sqrt{2}}^{\sqrt{2}} \pi[2-y^2]^2 \, dy$.

(3 marks)

(b) $\int \frac{dx}{(1+\sqrt{x})^3}$.

(5 marks)

(c) $\int \frac{2z \, dz}{\sqrt[3]{z^2+1}}$.

(5 marks)

(d) $\int e^x \cos x \, dx$.

(7 marks)

(e) $\int \frac{6x+7}{(x+2)^2} \, dx$.

(5 marks)

Q4 Answer all the following questions.

(a) Determine the area of the region between the x - axis and the graph of $f(x) = x^3 - x^2 - 2x$ for $-1 \leq x \leq 2$.

(9 marks)

(b) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$ for $1 \leq x \leq 2$, about the x - axis.

(8 marks)

(c) Calculate the length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1$ for $0 \leq x \leq 1$.

(8 marks)

- END OF QUESTIONS -

APPENDIX A

Table APPENDIX A.1

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x] = -\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\, x] = -\text{cosec}\, x \cot x$	$\int \text{cosec}\, x \cot x\,dx = -\text{cosec}\, x + C$
$\frac{d}{dx}[e^x] = e^x$	$\int e^x\,dx = e^x + C$

Table APPENDIX A.2

Trigonometric Identities
$\cos^2 x + \sin^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$\cot^2 x + 1 = \csc^2 x$
$\sin 2x = 2\sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$\cos 2x = 2\cos^2 x - 1$
$\cos 2x = 1 - 2\sin^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$
Logarithm
$a^x = e^{x \ln a}$
$\log_a x = \dfrac{\log_b x}{\log_b a}$

APPENDIX B

Area between two curves

Case 1- Integrating with respect to x: $A = \int_a^b [f(x) - g(x)]dx$

Case 2- Integrating with respect to y: $A = \int_c^d [f(y) - g(y)]dy$

Area of surface of revolution

Case 1- Revolving the portion of the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Case 2- Revolving the portion of the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve- Revolving the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Parametric curve- Revolving the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arc length

x-axis: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

y-axis: $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Curvature

Curvature, $\kappa = \dfrac{\left[\frac{d^2y}{dx^2}\right]}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{\kappa}$

Curvature of parametric curve

Curvature, $\kappa = \dfrac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{\kappa}$