CONFIDENTIAL

Universiti Tun Hussein Onn Malaysia

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER I
## SESSION 2023/2024

| | | |
|---|---|---|
| COURSE NAME | : | CIVIL ENGINEERING MATHEMATICS III/ ENGINEERING MATHEMATICS |
| COURSE CODE | : | BFC 24103 / BFC 25103 |
| PROGRAMME CODE | : | BFF |
| EXAMINATION DATE | : | JANUARY / FEBRUARY 2024 |
| DURATION | : | 3 HOURS |
| INSTRUCTIONS | : | 1. ANSWER ALL QUESTIONS<br>2. THIS FINAL EXAMINATION IS CONDUCTED VIA<br>☐ Open book<br>☒ Closed book<br>3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK |

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

CONFIDENTIAL

**Q1** (a) Find the 2nd partial derivatives of the equation and show that $f_{xyz}=f_{zyx}$.

$$f(x,y,z)=\sin(3x+yz)$$

(5 marks)

(b) Wave in Pantai Beroga is defined by the equation $f(x, y) = x^2 - 4xy + y^3 + 4y$. In developing the coastal structure, the local maximum or a local minimum, or the value of a saddle point need to be determined. Determine its critical points and determine whether the wave has a local maximum or a local minimum, or the value of a saddle point.

(10 marks)

(c) If $u=x^4y+y^2z^3$, where $x=rse^t$, $y=rs^2e^{-t}$ and $z=r^2s \sin t$, determine the value of $\frac{\delta u}{\delta s}$ when r=2, s=1, t=0.

(5 marks)

(d) Sketch the contour plot using three-level curves for $z=8-4x-2y$ for $z=-8, 0, 8$.

(5 marks)

**Q2** (a) Sketch a diagram and evaluate of the region bounded by:

line $x+y=8$ and curve $xy=5$

(5 marks)

(b) Use a triple integral to determine the volume of the region below $x = 3$ and $x = y^2 + z^2$.

(6 marks)

(c) In the design of a dam, the depth of hydrostatic pressure should be considered. The depth of hydrostatic pressure can be calculated by using the formula:

$$y_r = y_c + \frac{I_{xc}}{y_c A}$$

Where
$y_r$ = depth of hydrostatic pressure
$y_c$ = center of gravity
$I_{xc}$ = moment of inertia
A = area

By using this formula, calculate the depth of hydrostatic pressure of the hydraulic structure with surface $z = -y + 5$, planes $z = 0$, $y = 0$ and width 7 meters. Given that the density function is $(x, y, z) = z$.

(14 marks)

**Q3** Answer the following questions:

(a) Let $f(x, y, z) = x^2y - y^2z + xz^3$. Find the gradient at (1,2,1).

(4 marks)

(b) Given the vector field $F = (y^4 - x^2z^2)i + (x^2 + y^2)j - (x^2yz)k$

(i) Find the divergence of the vector field.

Hint: $div\,F = \nabla \cdot F$

(4 marks)

(ii) Find the curl of the vector field at point (1, 3, -2).

Hint: $curl\,F = \nabla \times F$

(6 marks)

(c) Evaluate the line integral $\int (x + y + z)ds$ where C is given by $x = 3\cos t,\ \ y = 3\sin t,\ \ z = 4t$ and with condition $0 \leq t \leq 2\pi$.

(6 marks)

(d) Calculate the surface integral $\iint 6xy\ dS$ where S is the portion of the plane $x + y + z = 1$ that lies in the first octant and is in the front of the yz-plane as shown in **Figure Q3.1**.

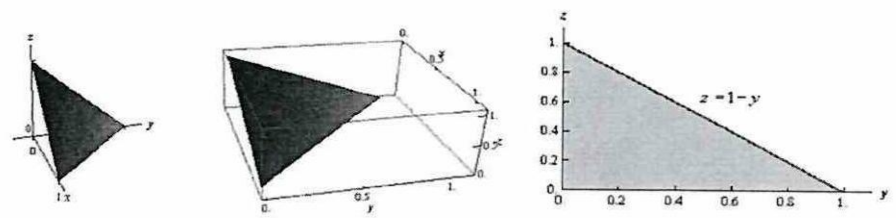


**Figure Q3.1**

(5 marks)

**Q4** Answer the following questions:

(a) Determine the Laplace transform of the equation $f(t) = (t + 2)^3$.

(4 marks)

(b) Apply Laplace transform to solve the initial value problem $y(0) = 0$ for the equation $y' + y = \cos t$ .

(10 marks)

(c) Analyse $\int_0^t (t-u)\sin 2u\, du$ to determine the Laplace transforms following the convolution theorem.

(6 marks)

(d) A 3 kg object is attached to the spring and will stretch the spring 392 mm by itself. There is no damping in the system and a forcing function of the form $F(t) = 10\cos(\omega t)$ is attached to the object, and the system will experience resonance. If the object is initially displaced 20 cm downward from its equilibrium position and given a velocity of 10 cm/sec upward. Solve for the general solution for the displacement, $u$.

Hint: $mu'' + ku = F(t)$

(5 marks)

**- END OF QUESTIONS -**

APPENDIX A

**Formula**

Characteristic Equation and General Solution

| Case | Roots of the Characteristic Equation | General Solution |
|---|---|---|
| 1 | $m_1$ and $m_2$ ; real and distinct | $y = Ae^{m_1 x} + Be^{m_2 x}$ |
| 2 | $m_1 = m_2 = m$ ; real and equal | $y = (A + Bx)e^{mx}$ |
| 3 | $m = \alpha \pm i\beta$ ; imaginary | $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$ |

Particular Integral of $ay'' + by' + cy = f(x)$ : Method of Undetermined Coefficients

| $f(x)$ | $y_p(x)$ |
|---|---|
| $P_n(x) = A_n x^n + \cdots + A_1 x + A_0$ | $x^r(B_n x^n + \cdots + B_1 x + B_0)$ |
| $Ce^{\alpha x}$ | $x^r(Pe^{\alpha x})$ |
| $C\cos\beta x$ or $C\sin\beta x$ | $x^r(p\cos\beta x + q\sin\beta x)$ |

Particular Integral of $ay'' + by' + cy = f(x)$ : Method of Variation of Parameters

| Wronskian | Parameter | Solution |
|---|---|---|
| $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$ | $u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$ | $y_p = u_1 y_1 + u_2 y_2$ |

Laplace Transforms

$$\mathbf{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

| $f(t)$ | $F(s)$ | $f(t)$ | $F(s)$ |
|---|---|---|---|
| $a$ | $\frac{a}{s}$ | $H(t-a)$ | $\frac{e^{-as}}{s}$ |
| $t^n$, $n = 1, 2, 3, \ldots$ | $\frac{n!}{s^{n+1}}$ | $f(t-a)H(t-a)$ | $e^{-as}F(s)$ |
| $e^{at}$ | $\frac{1}{s-a}$ | $\delta(t-a)$ | $e^{-as}$ |
| $\sin at$ | $\frac{a}{s^2+a^2}$ | $f(t)\delta(t-a)$ | $e^{-as}f(a)$ |
| $\cos at$ | $\frac{s}{s^2+a^2}$ | $\int_0^t f(u)g(t-u)\,du$ | $F(s).G(s)$ |
| $\sinh at$ | $\frac{a}{s^2-a^2}$ | $y(t)$ | $Y(s)$ |
| $\cosh at$ | $\frac{s}{s^2-a^2}$ | $\dot{y}(t)$ | $sY(s)-y(0)$ |
| $e^{at}f(t)$ | $F(s-a)$ | $\ddot{y}(t)$ | $s^2Y(s)-sy(0)-\dot{y}(0)$ |
| $t^n f(t)$, $n = 1, 2, 3, \ldots$ | $(-1)^n \frac{d^nF(s)}{ds^n}$ | | |

Periodic Function for Laplace transform : $\mathbf{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt, \quad s > 0.$

| **The denominator** | **Partial Fraction** |
|---|---|
| $ax+b$ | $\frac{A}{ax+b}$ |
| $(ax+b)^2$ | $\frac{A}{ax+b} + \frac{A_1}{ax+b^2}$ |
| $ax^2+bx+c$ | $\frac{Ax+B}{ax^2+bx+c}$ |
| $(ax^2+bx+c)^2$ | $\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2}$ |

| Case | $G(a,b)$ | Result |
|---|---|---|
| 1 | $G(a,b)>0$<br>$f_{xx}(a,b)<0$ | $f(x,y)$ has a local maximum value at $(a,b)$ |
| 2 | $G(a,b)>0$<br>$f_{xx}(a,b)>0$ | $f(x,y)$ has a local minimum value at $(a,b)$ |
| 3 | $G(a,b)<0$ | $f(x,y) has\ a\ saddle\ point\ at\ (a,b)$ |
| 4 | $G(a,b)=0$ | inconclusive |

Tangent Plane: $z - z_0 = f_x(x_o, y_0)(x - x_0) + f_y(x_o, y_0)(y - y_0)$

Local Extreme Value: $G(x,y) = f_{xx}(x,y) \times f_{yy}(x,y) - [f_{xy}(x,y)]^2$

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(\frac{y}{x})$ and

$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)d\ dz\ dr\ d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\theta$, $x^2 + y^2 + z^2 = \rho^2$, $0 \ll \theta \ll 2\pi$, $0 \ll \phi \ll \pi$ and $\iiint f(x,y,z)dV = \iiint f(\rho,\phi,\theta)\rho^2\sin\phi\ d\rho d\phi d\theta$

For lamina

Mass, $m = \iint_R \delta(x,y)\ dA$

Moment of mass: y-axis: $M_y = \iint_R x\delta(x,y)\ dA$ x-axis, $M_x = \iint_R y\delta(x,y)\ dA$

Center of mass, $(\bar{x}, \bar{y}) = (\frac{M_y}{m}, \frac{M_x}{m})$

Centroid for homogenous lamina: $\bar{x} = \frac{1}{area}\iint_R x\ dA$ $\bar{y} = \frac{1}{area}\iint_R y\ dA$

Moment inertia:

Y-axis: $I_y = \iint_R x^2\,\delta(x,y)\ dA$ x-axis: $I_x = \iint_R y^2\,\delta(x,y)\ dA$

Z-axis (or origin): $I_z = I_0 = \iint_R (x^2 + y^2)\,\delta(x,y)\ dA = I_x + I_y$

For solid

Mass, $m = \iiint_G \delta(x,y)\ dV$

Moment of mass:

yz-plane: $M_{yz} = \iiint_G x\,\delta(x,y,z)\ dV$

xz-plane: $M_{xz} = \iiint_G y\,\delta(x,y,z)\ dV$

xy-plane: $M_{xy} = \iiint_G z\,\delta(x,y,z)\ dV$

Center of gravity, $(\bar{x}, \bar{y}, \bar{z}) = (\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m})$

Moment inertia:

$$I_y = \iiint_G (x^2 + z^2)\, \delta(x, y, z)\, dV$$

$$I_x = \iiint_G (y^2 + z^2)\, \delta(x, y, z)\, dV$$

$$I_z = \iiint_G (x^2 + y^2)\, \delta(x, y, z)\, dV$$

Directional derivative: $D_u f(x, y) = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot u$

Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, then the divergence of $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The curl of

$$\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let $C$ is a smooth curve given by $r(t) = x(t)i + y(t)j + z(t)k$, $t$ is parameter, then

The unit tangent vector; $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The binormal vector: $B(t) = T(t) \times N(t)$

The curvature: $K = \frac{\mathbf{T}'(t)}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}' \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

The radius of curvature: $\rho = 1/K$

Gauss Theorem: $\iint_\sigma \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_G \nabla . \mathbf{F}\, dV$

Stokes Theorem: $\oint_C \mathbf{F} . d\mathbf{r} = \iint_\sigma (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$

Arc length, If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t\epsilon[a, b]$, then the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\|\, dt = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t\epsilon[a, b]$, then the arc length

$$s = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$$

**Trigonometric and Hyperbolic Identities**

| **Trigonometric** | **Hiperbolic** |
|---|---|
| $\cos^2 x + \sin^2 x = 1$ | $\sinh x = \dfrac{e^x - e^{-x}}{2}$ |
| $1 + \tan^2 x = \sec^2 x$ | $\cosh x = \dfrac{e^x + e^{-x}}{2}$ |
| $\cot^2 x + 1 = \operatorname{cosec}^2 x$ | $\cosh^2 x - \sinh^2 x = 1$ |
| $\sin 2x = 2\sin x \cos x$ | $1 - \tanh^2 x = \operatorname{sech}^2 x$ |
| $\cos 2x = \cos^2 x - \sin^2 x$ | $\coth^2 x - 1 = \operatorname{cosech}^2 x$ |
| $\cos 2x = 2\cos^2 x - 1$ | $\sinh 2x = 2\sinh x \cosh x$ |
| $\cos 2x = 1 - 2\sin^2 x$ | $\cosh 2x = \cosh^2 x + \sinh^2 x$ |
| $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$ | $\cosh 2x = 2\cosh^2 x - 1$ |
| $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$ | $\cosh 2x = 1 + 2\sinh^2 x$ |
| $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$ | $\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$ |
| $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$ | $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$ |
| $2\sin x \cos y = \sin(x + y) + \sin(x - y)$ | $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$ |
| $2\sin x \sin y = -\cos(x + y) + \cos(x - y)$ | $\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$ |
| $2\cos x \cos y = \cos(x + y) + \cos(x - y)$ | |