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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2023/2024

COURSE NAME	:	NUMERICAL METHOD
COURSE CODE	:	BFC25203
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JANUARY/ FEBRUARY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF SEVEN (7) PAGES

Q1 Interpolation and numerical differentiation are often performed using civil engineering software to optimize the design and construction of infrastructure project.

(a) Prepare divided difference table of road alignment profile for a road design project that using the data given in **Table Q1.1**. Then, transform the data from divided difference table obtained into equation of $S(x)$ using natural cubic spline polynomial. Do all calculations in 3 decimal places.

Table Q1.1 Road alignment profile for a road design project

Distance, x (km)	1	2	3	4
Road alignment, $f(x)$ (%)	1	1	0	-1

(15 marks)

(b) **Table Q1.2** shows the flow of stormwater in drainage system in Taman University. As a civil engineer, you are assigned to solve flash flood problem due to extreme increasing water level in this residential area. Do all calculations in 3 decimal places.

Table Q1.2 Tabulates the velocity, v of an object at various points in time, t.

Time, t (minute)	1.3	1.4	1.5	1.6	1.7	1.8
Velocity, v (m/min)	1.418	1.715	2.037	2.336	2.746	3.099

(i) Determine velocity of stormwater at t=1.5 minutes using two (2) point and three (3) point forward difference formula with h=0.1 minutes.

(6 marks)

(ii) Given that acceleration of stormwater, $v = x''(t)$. By taking h=0.2 minutes, estimate the acceleration of stormwater at time t=1.6 by using appropriate difference formula.

(4 marks)

Q2 Civil engineers use numerical integration to determine property boundary determination and land subdivision by using GPS data. You have been selected to join a land survey company to solve the following situations:

(a) Estimate the area of land plot with a =0, b= 1 by using suitable Simpson's rule with number of divisions, n = 9. Do all calculations in 4 decimal places.

$$\int_0^1 \frac{dx}{\sqrt{1+x^4}}$$

(10 marks)

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(b) Calculate another land region by using 2-point and 3-point Gauss quadrature. Do all calculations in 4 decimal places.

$$\int_{1}^{3} \frac{x^2}{1+x^4}\, dx$$

(15 marks)

Q3 The stability of the bridge construction can be calculated and determined by the natural frequency of a bridge system (smallest magnitude eigenvalue) and its corresponding eigenvector in the matrix form as:

$$C = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 2 & 0 \\ 2 & -2 & 4 \end{bmatrix}$$

Use $v^{(0)} = (1 \quad 0 \quad 1)$ and stop the iteration until $|m_{k+1} - m_k| < 0.005$. Do your calculations in 3 decimal places.

(25 marks)

Q4 Solve the following problems.

(a) Consider the equation of dynamic response, v' of a house in Kota Kinabalu when subjected to the earthquake is as follow:

$$v' = \frac{F}{m} \cdot c - kv$$

where F is the seismic force, c is the damping coefficient, k is the stiffness of the structure and m is the mass of the house. Assume the following values:

- seismic force, F = 100 kN,
- mass of the house, m = 100 times of your body weight (in kg)
- damping coefficient, c = your last digit of matric number (for example: AF02200*c*). If the last digit of your number is zero (0) then take = 1.
- Stiffness of the structure house, k = 5 kN/m
- Initial condition $v\ (0) = 0\ m/s$

(i) Estimate the dynamic motion, v' due to the earthquake at time t = 4 seconds using the fourth-order Runge Kutta Mehtod with $\Delta t = 1$ second and $v_0 = 0$. Do all calculations in 3 decimal places.

(10 marks)

(ii) Assume that the displacement of the house with the increment of the dynamic motion for more than 25 % can lead to structural damage and collapse. Based on the answer in Q4 (a) (i), is this house still safe? Justify your answer.

(2 marks)

(b) Given the heat equation use for assessing the heat gain of a building wall as follow:

$$\frac{\partial u}{\partial t} = 6\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 4, t > 0$$

with boundary condition $u(0,t) = 20\ °C$ and $u(4,t) = 22 - e^t$ for $t > 0$. The initial condition $u(x,0) = 20 + \sin(2x)$ for $0 \le x \le 4$. Using implicit Crank-Nicolson method, determine the heat equation at the first level only ($t \le 0.5$) by taking $\Delta x = h = 1.0$ and $\Delta t = k = 0.5$. Do all calculations in 3 decimal places.

(13 marks)

- END OF QUESTIONS -

APPENDIX A

Nonlinear equations

Lagrange Interpolating : $L_i = \frac{(x-x_1)(x-x_2)}{(x_i-x_1)(x_i-x_2)} \cdots \frac{(x-x_n)}{(x_i-x_n)}$; $f(x) = \sum_{i=1}^{n} L_i(x) f(x_i)$

Newton-Raphson Method : $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0,1,2 \ldots.$

System of linear equations

Gauss-Seidel Iteration:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}\ x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\ x_j^{(k)}}{a_{ii}}, \forall i = 1,2,3, \ldots, n.$$

Interpolation

Natural Cubic Spline:

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\}, k = 0,1,2,3, \ldots, n-1$$

$$b_k = 6(d_{k+1} - d_k), k = 0,1,2,3, \ldots, n-2,$$

When; $m_0 = 0, m_n = 0,$

$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, k = 0,1,2,3, \ldots, n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6} h_k\right)(x_{k+1} - x)$$
$$+ \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6} h_k\right)(x - x_k) \quad , k = 0,1,2,3, \ldots n-1$$

Numerical Differentiation

2-point forward difference: $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x) - f(x-h)}{h}$

3-point central difference: $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x)-4f(x-h)+f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

3-point central difference: $f''(x) \approx \frac{f(x+h)-2f(x)+f(x-h)}{h^2}$

5-point difference formula: $f''(x) \approx \frac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$

Numerical Integration

Simpson $\frac{1}{3}$ Rule : $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\ odd}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\ even}}^{n-2} f_i\right]$

Simpson $\frac{3}{8}$ Rule : $\int_a^b f(x)dx \approx \frac{3}{8}h\,[(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \cdots + f_{n-3}]$

2-point Gauss Quadrature: $\int_a^b g(x)dx = \left[g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)\right]$

3-point Gauss Quadrature: $\int_a^b g(x)dx = \left[\frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)\right]$

Eigen Value

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}Av^{(k)}, k = 0,1,2 ....$

Shifted Power Method :$v^{(k+1)} = \frac{1}{m_{k+1}}A_{shifted}v^{(k)}, k = 0,1,2 ....$

Ordinary Differential Equation

Fourth-order Runge-Kutta Method : $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $k_4 = hf(x_i + h, y_i + k_3)$

Partial Differential Equation

Heat Equation: Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i.j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Heat Equation: Crank-Nicolson Implicit Finite-Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}}$$

$$\frac{u_{i,j+1} - u_{i.j}}{k} = \frac{c^2}{2}\left(\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{h^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}\right)$$

Poisson Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i.j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Wave Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j-1} - 2u_{i.j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$