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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2023/2024

COURSE NAME : CIVIL ENGINEERING MATHEMATICS IV

COURSE CODE : BFC 24203

PROGRAMME CODE : BFF

EXAMINATION DATE : JANUARY / FEBRUARY 2024

DURATION : 3 HOURS

INSTRUCTIONS :
1. ANSWER ALL QUESTIONS
2. THIS FINAL EXAMINATION IS CONDUCTED VIA
   ☐ Open book
   ☒ Closed book
3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 Interpolation and numerical differentiation are often performed using civil engineering software to optimize the design and construction of infrastructure project.

(a) Prepare divided difference table of road alignment profile for a road design project that using the data given in **Table Q1.1**. Then, transform the data from divided difference table obtained into equation of $S(x)$ using natural cubic spline polynomial. Do all calculations in 3 decimal places.

Table Q1.1 Road alignment profile for a road design project

Distance, x (km)	1	2	3	4
Road alignment, $f(x)$ (%)	1	1	0	-1

(15 marks)

(b) **Table Q1.2** shows the flow of stormwater in drainage system in Taman University. As a civil engineer, you are assigned to solve flash flood problem due to extreme increasing water level in this residential area. Do all calculations in 3 decimal places.

Table Q1.2 Tabulates the velocity, v of an object at various points in time, t.

Time, t (minute)	1.3	1.4	1.5	1.6	1.7	1.8
Velocity, v (m/min)	1.418	1.715	2.037	2.336	2.746	3.099

(i) Determine velocity of stormwater at t=1.5 minutes using two (2) point and three (3) point forward difference formula with h=0.1 minutes.

(6 marks)

(ii) Given that acceleration of stormwater, $v = x''(t)$. By taking h=0.2 minutes, estimate the acceleration of stormwater at time t=1.6 by using appropriate difference formula.

(4 marks)

Q2 Civil engineers use numerical integration to determine property boundary determination and land subdivision by using GPS data. You have been selected to join a land survey company to solve the following situations:

(a) Estimate the area of land plot with a =0, b= 1 by using suitable Simpson's rule with number of divisions, n = 9. Do all calculations in 4 decimal places.

$$\int_0^1 \frac{dx}{\sqrt{1+x^4}}$$

(10 marks)

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(b) Calculate another land region by using 2-point and 3-point Gauss quadrature. Do all calculations in 4 decimal places.

$$\int_{1}^{3} \frac{x^2}{1+x^4} \, dx$$

(15 marks)

Q3 The stability of the bridge construction can be calculated and determined by the natural frequency of a bridge system (smallest magnitude eigenvalue) and its corresponding eigenvector in the matrix form as:

$$C = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 2 & 0 \\ 2 & -2 & 4 \end{bmatrix}$$

Use $v^{(0)} = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$ and stop the iteration until $|m_{k+1} - m_k| < 0.005$. Do your calculations in 3 decimal places.

(25 marks)

Q4 Solve the following problems.

(a) Consider the equation of dynamic response, v' of a house in Kota Kinabalu when subjected to the earthquake is as follow:

$$v' = \frac{F}{m} \cdot c - kv$$

where F is the seismic force, c is the damping coefficient, k is the stiffness of the structure and m is the mass of the house. Assume the following values:

- seismic force, F = 100 kN,
- mass of the house, m = 100 times of your body weight (in kg)
- damping coefficient, c = your last digit of matric number (for example: AF02200c). If the last digit of your number is zero (0) then take = 1.
- Stiffness of the structure house, k = 5 kN/m
- Initial condition $v(0) = 0 \, m/s$

(i) Estimate the dynamic motion, v' due to the earthquake at time t = 4 seconds using the fourth-order Runge Kutta Mehtod with $\Delta t = 1$ second and $v_0 = 0$. Do all calculations in 3 decimal places.

(10 marks)

(ii) Assume that the displacement of the house with the increment of the dynamic motion for more than 25 % can lead to structural damage and collapse. Based on the answer in Q4 (a) (i), is this house still safe? Justify your answer.

(2 marks)

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(b) Given the heat equation use for assessing the heat gain of a building wall as follow:

$$\frac{\partial u}{\partial t} = 6 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 4, t > 0$$

with boundary condition $u(0, t) = 20\ °C$ and $u(4, t) = 22 - e^t$ for $t > 0$. The initial condition $u(x, 0) = 20 + \sin(2x)$ for $0 \le x \le 4$. Using implicit Crank-Nicolson method, determine the heat equation at the first level only ($t \le 0.5$) by taking $\Delta x = h = 1.0$ and $\Delta t = k = 0.5$. Do all calculations in 3 decimal places.

(13 marks)

- END OF QUESTIONS -

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APPENDIX A

Nonlinear equations

Lagrange Interpolating : $L_i = \frac{(x-x_1)(x-x_2)}{(x_i-x_1)(x_i-x_2)} \cdots \frac{(x-x_n)}{(x_i-x_n)}; f(x) = \sum_{i=1}^{n} L_i(x)f(x_i)$

Newton-Raphson Method : $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0,1,2 \dots$

System of linear equations

Gauss-Seidel Iteration:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} \, x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} \, x_j^{(k)}}{a_{ii}}, \forall i = 1,2,3,\dots,n.$$

Interpolation

Natural Cubic Spline:

$$\left. \begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1}-f_k}{h_k} \end{aligned} \right\}, k = 0,1,2,3,\dots,n-1$$

$$b_k = 6(d_{k+1} - d_k), k = 0,1,2,3,\dots,n-2,$$

When; $m_0 = 0, m_n = 0,$

$$h_k m_k + 2(h_k + h_{k+1})m_{k+1} + h_{k+1}m_{k+2} = b_k, k = 0,1,2,3,\dots,n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1} - x)$$

$$+ \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x - x_k) \quad , k = 0,1,2,3,\dots n-1$$

Numerical Differentiation

2-point forward difference: $f'(x) \approx \frac{f(x+h)-f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x)-f(x-h)}{h}$

3-point central difference: $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x)-4f(x-h)+f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

3-point central difference: $f''(x) \approx \frac{f(x+h)-2f(x)+f(x-h)}{h^2}$

5-point difference formula: $f''(x) \approx \frac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$

Numerical Integration

Simpson $\frac{1}{3}$ Rule : $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\ odd}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\ even}}^{n-2} f_i\right]$

Simpson $\frac{3}{8}$ Rule : $\int_a^b f(x)dx \approx \frac{3}{8}h\left[(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \cdots + f_{n-3}\right]$

2-point Gauss Quadrature: $\int_a^b g(x)dx = \left[g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)\right]$

3-point Gauss Quadrature: $\int_a^b g(x)dx = \left[\frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)\right]$

Eigen Value

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}Av^{(k)}, k = 0,1,2 \dots$

Shifted Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}A_{shifted}v^{(k)}, k = 0,1,2 \dots$

Ordinary Differential Equation

Fourth-order Runge-Kutta Method : $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $k_4 = hf(x_i + h, y_i + k_3)$

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Partial Differential Equation

Heat Equation: Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Heat Equation: Crank-Nicolson Implicit Finite-Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}}$$

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{c^2}{2}\left(\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{h^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}\right)$$

Poisson Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Wave Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

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