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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2023/2024

COURSE NAME	:	ACTUARIAL MATHEMATICS I
COURSE CODE	:	BWA 31403
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2024
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 The term, whole life and endowment insurance are traditional products, providing cash benefits on death or maturity, usually with predetermined premium and benefit amounts.

(a) Define

(i) term insurance;

(2 marks)

(ii) whole life insurance;

(2 marks)

(iii) endowment insurance.

(3 marks)

(b) Ali has just bought life insurance from XYZ Company, where the benefit is payable upon death. List the major risks for the company. Eplain your answer.

(3 marks)

Q2 Actuarial science has developed its own notation for survival and mortality probabilities. Formulate the meaning of these sentences to actuarial notations:

(a) The probability that (30) dies before age 40.

(2 marks)

(b) The probability that (30) survives to at least age 40.

(2 marks)

(c) The probability that (30) dies between the ages of 35 and 40.

(3 marks)

(d) The probability that a life currently aged 30 who was selected a year ago will die next year.

(3 marks)

Q3 For each of the whole life, term and endowment insurance benefits, we identify the random variables representing the present values of the benefits and we derive expressions for moments of these random variables.

(a) ABC Inc. proposes an insurance where the present values of the benefits are given below:

$$Z_1 = \begin{cases} 20\upsilon^t, & t \leq 15, \\ 10\upsilon^t, & t > 15. \end{cases} \qquad Z_2 = \begin{cases} 0, & t \leq 5, \\ 10\upsilon^t, & 5 < t \leq 15, \\ 10\upsilon^{15}, & t > 15. \end{cases}$$

(i) Write down the formula for the expected values of Z_1 and Z_2 in integral form.

(4 marks)

(ii) Construct expressions in terms of standard actuarial functions for the expected values of Z_1 and Z_2.

(4 marks)

(b) Muthu is an 80-year-old man who buys a whole life insurance policy. The policy will pay RM 70,000 at the end of the year of his death. However, the insurance benefit will only be given if he dies at least five years following policy issue. Based on **Table Q3.1,** estimate the actuarial present value of this life insurance where $i = 6\%$.

Table Q3.1 Life Table

x	80	81	82	83	84	85	86
l_x	250	217	161	107	62	28	0

(6 marks)

(c) Ana and Liza buy a whole life policy insurance on the day of their birthdays. Both policies will pay RM 50,000 at the end of the year of death. Ana is 45 years old and the actuarial present value of her insurance is RM 25,000. Liza is one year younger than Ana and the actuarial present value of her insurance is RM 23,702. With $i = 6\%$, calculate the probability that Liza will die within one year.

(6 marks)

Q4 An annuity is a benefit in the form of a regular series of payments, usually conditional on the survival of the policyholder.

(a) Given that $\ddot{a}_{50:\overline{10|}} = 8.2066$, $a_{50:\overline{10|}} = 7.8277$ and ${}_{10}p_{50} = 0.9195$. Find the force of interest.

(4 marks)

(b) **Table Q4.1** shows the Illustrative Life Table. If Chong, at age 60, buys a whole life immediate annuity of RM 3,000 per annum with a 6% annual interest rate, compute the actuarial present value of the annuity.

Table Q4.1 Illustrative Life Table

Age	$\ddot{a}_x$	$1000A_x$	$1000\left({}^2A_x\right)$
60	11.14535	369.1310	177.4113
61	10.90142	382.7858	188.1682
62	10.65836	396.6965	199.4077
63	10.40837	410.8471	211.1318
64	10.15444	425.2202	223.3401
65	9.89693	439.7965	236.0299

(4 marks)

(c) 80-year-old Sarah buys a whole life immediate annuity, which will pay RM 60,000 at the end of the year. Suppose that $i = 6\%$. By referring to **Table Q4.2,** construct the single benefit premium for this annuity.

Table Q4.2 Life Table

x	80	81	82	83	84	85	86
l_x	250	217	161	107	62	28	0

(6 marks)

(d) Ahmad, at 60 years old, saves RM 750,000 in his retirement account. IOP Inc. offers Ahmad a whole life annuity-due which pays RM P at the beginning of the year while he is alive for the same amount as his savings. The annuity is priced assuming that $i = 6.5\%$. The company charges Ahmad 20% more of the actuarial present value of the annuity. Given $A_x = 23.766/x$, generate P.

(6 marks)

Q5 Insurance is issued to Rina, who is aged 40. If Rina dies within five years, a death benefit of RM 6,000 is payable at the end of the year of death. Otherwise, a sum of RM 2,500 is payable yearly in advance from the age of 45.

(a) By using **Table Q5.1** with 6% annual rate of interest, calculate the net single premium for the insurance.

Table Q5.1 Illustrative Life Table

Age	l_x	$\ddot{a}_x$	$1000A_x$
40	93 131.64	14.81661	161.3242
41	92 872.62	14.68645	168.6916
42	92 595.70	14.55102	176.3572
44	91 981.47	14.26394	192.6071
45	91 640.50	14.11209	201.2024
46	91 274.25	13.95459	210.1176
47	90 880.48	13.79136	219.3569
48	90 456.78	13.62235	228.9234
49	90 000.55	13.44752	238.8198

(12 marks)

(b) Rina then changes her mind and instead takes out a 5-year pure endowment insurance of RM 8,500. Based on 6% annual rate of interest, propose the annual premium.

(8 marks)

- **END OF QUESTIONS -**

APPENDIX A

LIST OF FORMULAS

$$F_x(t)=\frac{F_0(x+t)-F_0(x)}{S_0(x)} \qquad S_0(x+t)=S_0(x)S_x(t) \qquad \mu_{x+t}=-S_x'(t)/S_x(t)$$

$${}_tp_x=\frac{l_{x+t}}{l_x} \quad d_x=l_x-l_{x+1} \quad d_x=l_xq_x \quad {}_{t|u}q_x={}_tp_x-{}_{t+u}p_x \quad d=i\upsilon$$

$$A_{x:\overline{n}|}^{\ \ 1}={}_nE_x=\upsilon^n{}_np_x \qquad A_{x:\overline{n}|}^{\ \ 1}=\int_n^\infty \upsilon^n\,{}_tp_x\,\mu_{x+t}\,dt \qquad \overline{A}_{x:\overline{n}|}^{1}=\int_0^n \upsilon^t\,{}_tp_x\,\mu_{x+t}\,dt \qquad \overline{A}_x=\int_0^\infty \upsilon^t\,{}_tp_x\,\mu_{x+t}\,dt$$

$$A_x=\textstyle\sum_{k=0}^{\infty}\upsilon^{k+1}\,{}_{k|}q_x \qquad A_{x:\overline{n}|}^{1}=\sum_{k=0}^{n-1}\upsilon^{k+1}\,{}_{k|}q_x \qquad {}_{u|}\bar{A}_x=\int_u^\infty \upsilon^t\,{}_tp_x\,\mu_{x+t}\,dt=\bar{A}_x-\bar{A}_{x:\overline{u}|}^{1}$$

$$\overline{A}_{x:\overline{n}|}=\overline{A}_{x:\overline{n}|}^{1}+\overline{A}_{x:\overline{n}|}^{\ \ 1} \qquad A_{x:\overline{n}|}=A_{x:\overline{n}|}^{1}+A_{x:\overline{n}|}^{\ \ 1} \qquad {}_{u|}\overline{A}_{x:\overline{n}|}^{1}=\int_u^{u+n}\upsilon^t\,{}_tp_x\,\mu_{x+t}\,dt=\overline{A}_{x:\overline{u+n}|}^{1}-\overline{A}_{x:\overline{u}|}^{1}$$

$$A_{x:\overline{n}|}^{1}=A_x-\upsilon^n\,{}_np_x\,A_{x+n} \qquad A_x=\upsilon q_x+\upsilon p_xA_{x+1} \qquad \overline{A}_x=\frac{i}{\delta}A_x$$

$$\overline{a}_x=\frac{1-\overline{A}_x}{\delta} \qquad \overline{a}_x==\int_0^\infty \upsilon^t\,{}_tp_x\,dt \qquad \overline{a}_{x:\overline{n}|}=\frac{1-\overline{A}_{x:\overline{n}|}}{\delta} \qquad \overline{a}_{x:\overline{n}|}=\int_0^n \upsilon^t\,{}_tp_x\,dt$$

$$\ddot{a}_x=\frac{1-A_x}{d} \qquad \ddot{a}_x=\sum_{k=0}^{\infty}\upsilon^k\,{}_kp_x \qquad \ddot{a}_{x:\overline{n}|}=\frac{1-A_{x:\overline{n}|}}{d} \qquad \ddot{a}_{x:\overline{n}|}=\sum_{k=0}^{n-1}\upsilon^k\,{}_kp_x$$

$$a_x=\sum_{k=1}^{\infty}\upsilon^k\,{}_kp_x \qquad a_x=\ddot{a}_x-1 \qquad a_{x:\overline{n}|}=\sum_{k=1}^{n}\upsilon^k\,{}_kp_x \qquad \ddot{a}_{x:\overline{n}|}-a_{x:\overline{n}|}=1-\upsilon^n\,{}_np_x$$

$${}_{n|}\overline{a}_x=\int_n^\infty \upsilon^t\,{}_tp_x\,dt \qquad {}_{n|}\overline{a}_x={}_nE_x\,\overline{a}_{x+n} \qquad {}_{n|}\ddot{a}_x=\sum_{k=n}^{\infty}\upsilon^k\,{}_kp_x \qquad {}_{n|}\ddot{a}_x={}_nE_x\,\ddot{a}_{x+n}$$

$$P_x=\frac{A_x}{\ddot{a}_x} \qquad P_{x:\overline{n}|}^{1}=\frac{A_{x:\overline{n}|}^{1}}{\ddot{a}_{x:\overline{n}|}} \qquad P_{x:\overline{n}|}=\frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \qquad P_{x:\overline{n}|}^{\ \ 1}=\frac{A_{x:\overline{n}|}^{\ \ 1}}{\ddot{a}_{x:\overline{n}|}}$$