UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2023/2024

COURSE NAME	:	CALCULUS
COURSE CODE	:	DAS 20803
PROGRAMME CODE	:	DAU
EXAMINATION DATE	:	JANUARY/FEBRUARY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) The piecewise function $f(x)$ is given by the following:

$$f(x)=\begin{cases} \dfrac{1}{x+1}-3 & ; \quad -1<x \\ x^2+1 & ; \quad -1\le x\le 2 \\ 3-x & ; \quad x>2 \end{cases}.$$

(i) Sketch the graph of the functions.

(7 marks)

(ii) Determine the domain and range.

(2 marks)

(b) Given that $f(x)=2e^x+3x^2$, $g(x)=\dfrac{1}{2x+4}$ and $h(x)=cx+5$. Calculate:

(i) $f\circ g\left(\dfrac{3}{2}\right)$.

(3 marks)

(ii) inverse of $h(x)$.

(3 marks)

(iii) the value of $c$if $h^{-1}(4)=\dfrac{1}{4}$.

(2 marks)

(iv) $g\circ h\circ f(x)$.

(3 marks)

Q2 A function is written as follow:

$$f(x)=\begin{cases} 4x-3c & \text{if} \quad x\le 0 \\ \dfrac{1}{2x-1} & \text{if} \quad 0<x\le 1 \\ 2x^2-4x-3 & \text{if} \quad x>1 \end{cases}.$$

(a) Evaluate the following limit if it exists:

(i) $\lim\limits_{x\to 0^+} f(x)$.

(1 mark)

(ii) $\lim\limits_{x\to 0^-} f(x)$.

(1 mark)

(iii) $\lim\limits_{x\to 1} f(x)$.

(3 marks)

(b) Calculate the value of c such that $\lim_{x \to 0} f(x)$ exist.

(2 marks)

(c) Evaluate the following limits:

(i) $\lim_{x \to 1} \dfrac{x^3 - x}{x-1}$.

(3 marks)

(ii) $\lim_{x \to 9} \left(\dfrac{x-9}{\sqrt{x}-3} \right) - 2$.

(5 marks)

(d) Determine whether the following function is continuous at $t = 4$:

$$s(t) = \begin{cases} 3t^2 + 2t - 3 & ; \quad -1 \le t \le 4 \\ \dfrac{t^2 - 3t - 4}{16 - t^2} & ; \quad t > 4 \end{cases}.$$

(5 marks)

Q3 (a) Find the derivative for $x - 3xy - 4x = 2$ using implicit differentiation method.

(3 marks)

(b) Using parametric differentiation, evaluate the differential if $x = 3t - 4\sin t$ and $y = t^2 + t\cos \pi t$.

(3 marks)

(c) Given a function as follows:

$$y = \frac{1}{3}x^3 + 2x^2 - 12x + 3.$$

(i) Identify the local extreme and fill out the table **Table Q3(c)(i)**.

(8 marks)

(ii) Hence, sketch the graph.

(3 marks)

(d) Calculate the limit of the following function using L'Hópital Rule:

$$\lim_{x \to 2} \frac{2x^2 + 4x - 16}{4x^2 - 6x - 4}.$$

(3 marks)

Q4 (a) Solve the following integral using partial fraction:

$$\int_{6}^{8} \frac{4x+22}{x^2-3x-10}\, dx .$$

(6 marks)

(b) Find the integral:

$$\int \frac{\sin(\ln 3x)+x^2 \sin 3x}{x}\, dx .$$

(9 marks)

(c) Evaluate the integral below using Trapezoidal Rule with $n=4$. Give the answer to three decimal places.

$$\int_{2}^{4} \frac{x^2}{\sqrt{x-1}}\, dx .$$

(5 marks)

Q5 (a) A region **R** is bounded by the curve $y=x^2+5$ and $y=3x+5$ that intersect at point S and T as shown in **Figure Q5(a)**.

(i) Determine points S and T.

(4 marks)

(ii) Find the area of region **R**.

(4 marks)

(iii) Using cylindrical shells method, calculate the volume of region **R** that revolved about the y-axis.

(5 marks)

(b) Calculate the length of the curve $y=\dfrac{4\left(x^2-2\right)^{\frac{3}{2}}}{12}$ from $x=3$ to $x=6$.

(7 marks)

– END OF QUESTIONS –

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Table Q3(c)(i)

Critical and inflection points							
Test value							
f' behaviour							
f'' behaviour							

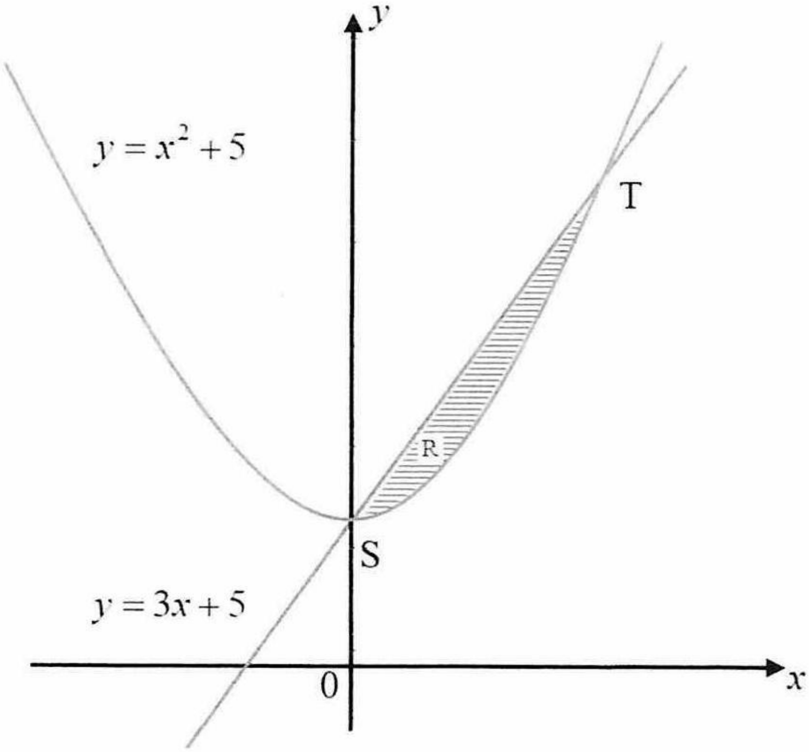

Figure Q5(a)

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LIST OF FORMULA

Table 1: Differentiation

$\frac{d}{dx}[c]=0$	$\frac{d}{dx}[\sin u]=\cos u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos u]=-\sin u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[cu]=c\frac{du}{dx}$	$\frac{d}{dx}[\ln\|u\|]=\frac{1}{u}\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[u\pm v]=\frac{du}{dx}\pm\frac{dv}{dx}$	$\frac{d}{dx}\left[e^u\right]=e^u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[uv]=u\frac{dv}{dx}+v\frac{du}{dx}$	$\frac{d}{dx}[a^u]=a^u\ln a\left(\frac{du}{dx}\right)$
$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}[\tan u]=\sec^2 u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[\log_a u]=\frac{1}{u}\log_b e\left(\frac{du}{dx}\right)$	$\frac{d}{dx}[\sec u]=\sec u\tan u\left(\frac{du}{dx}\right)$
Chain Rule: $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$	Parametric Differentiation: $\frac{dy}{dx}=\left(\frac{dy}{dt}\Big/\frac{dx}{dt}\right)=\frac{dy}{dt}.\frac{dt}{dx}$

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Table 2: Integration

$\int a\,dx = ax + C$	$\int \sin nx\,dx = -\frac{1}{n}(\cos nx) + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ (n \neq -1)$	$\int \cos nx\,dx = \frac{1}{n}(\sin nx) + C$
$\int \frac{1}{nx+b}\,dx = \frac{1}{n}\ln\lvert nx+b\rvert + C$	$\int \tan x\,dx = \ln\lvert \sec x \rvert + C$
$\int \frac{1}{b-nx}\,dx = -\frac{1}{n}\ln\lvert b-nx\rvert + C$	$\int \sec^2 x\,dx = \tan x + C$
$\int e^{nx}\,dx = \frac{1}{n}\left(e^{nx}\right) + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int e^{nx+b}\,dx = \frac{1}{n}e^{nx+b} + C$	$\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert + C$
Integration part by part: $\int u\,dv = uv - \int v\,du$	
Improper Integral: $\int_a^{\infty} f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$	
Identity: $1 + \tan^2 x = \sec^2 x$	

Area of Region

$$A = \int_a^b \left[f(x) - g(x)\right]dx \quad \text{or} \quad A = \int_c^d \left[w(y) - v(y)\right]dy$$

Volume Cylindrical Shells

$$V = \int_a^b 2\pi x\left[f(x) - g(x)\right]dx \quad \text{or} \quad V = \int_c^d 2\pi y\left[w(y) - v(y)\right]dy$$

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Arc Length

$$L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \quad \text{or} \quad L=\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

Partial Fraction

$$\frac{a}{x^2-1}=\frac{A}{x+1}+\frac{B}{x-1}$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[\big(f(a)+f(b)\big)+4\sum_{\substack{i=1\\ i\text{ odd}}}^{n-1} f(a+ih)+2\sum_{\substack{i=2\\ i\text{ even}}}^{n-2} f(a+ih)\right]; \quad n=\frac{b-a}{h}$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{i=1}^{n-1} f(a+ih)\right]; \quad n=\frac{b-a}{h}$$