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# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER I
## SESSION 2023/2024

| | | |
|---|---|---|
| COURSE NAME | : | TECHNICAL MATHEMATICS III |
| COURSE CODE | : | DAS 21002 |
| PROGRAMME CODE | : | DAK |
| EXAMINATION DATE | : | JANUARY/FEBRUARY 2024 |
| DURATION | : | 2 HOURS AND 30 MINUTES |
| INSTRUCTIONS | : | 1. ANSWER ALL QUESTIONS<br>2. THIS FINAL EXAMINATION IS CONDUCTED VIA<br>☐ Open book<br>☒ Closed book<br>3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK |

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES



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**Q1** (a) (i) Based on parametric equation of a line below, find point $(x_0, y_0, z_0)$ and the vector that is parallel to the line:

$$x = 2 - t$$
$$y = -3 + 5t \, .$$
$$z = 4 + 3t$$

(2 marks)

(ii) Express your answer in vector equation and symmetric equation.

(2 marks)

(b) Find equation of a plane consist of points $A(1,3,-1)$, $B(-1,-1,2)$, and $C(-2,0,4)$.

(6 marks)

(c) Given $z = 8(\cos 150° + i \sin 150°)$. By using De Moivre's theorem;

(i) Evaluate $z^3$.

(2 marks)

(ii) Determine $\sqrt[3]{z}$ and write your answer in $a+bi$ form. Plot the graph with three angles.

(8 marks)

**Q2** (a) **Table Q2(a)** shows the time in (minutes) recorded for a group of teenagers to complete a 5km run.

**Table Q2(a)**

| Time in (minutes) | Number of Teenagers |
|---|---|
| 16 - 20 | 62 |
| 21 – 25 | 88 |
| 26 – 30 | 16 |
| 31 – 35 | 13 |
| 36 – 40 | 11 |
| 41 – 45 | 10 |

If $x$ is the midpoint and $f$ is the frequency, construct a table that contain class limit, lower boundary, $x$, $x^2$, $f$, cumulative frequency, $fx$, $fx^2$, $\sum f$, $\sum f_i x_i$ and $\sum f_i x_i^2$.

(8 marks)

(b) Find the mean, median, mode, variance, and standard deviation for the time to complete the run.

(12 marks)

**Q3** (a) In an engineering course in a college, 90% of the students got A in Mathematics, 95% of the students got A in Physics, and 88% of the students got A for Mathematics and Physics.

(i) State whether the above situation is mutually exclusive or not. Give your reason.

(2 marks)

(ii) Draw a Venn diagram.

(4 marks)

(iii) By using the Venn diagram, shade and state the probability that the students got A in Mathematics or Physics.

(2 marks)

(iv) By using the Venn Diagram, shade and state the probability that the students do not get an A for neither Mathematics nor Physics.

(2 marks)

(b) **Table Q3(b)** shows the responses of 200 randomly sampled trainee teachers. The survey is about their willingness to be posted to East Malaysia as trained teachers upon graduation. A trainee is randomly chosen.

**Table Q3(b)**

| Posting to East Malaysia | Male | Female |
|---|---|---|
| Yes | 65 | 35 |
| No | 17 | 83 |

(i) Find the probability that male trainee does not agree to be posted to East Malaysia.

(2 marks)

(ii) Calculate the probability that trainee agrees to be posted to East Malaysia.

(2 marks)

(i) Compute the probability that trainee agrees to be posted to East Malaysia given that the trainee is female.

(3 marks)

(iv) Determine whether the events 'female' and 'does not agree to be posted to East Malaysia' are independent. Explain your answer.

(3 marks)

**Q4** (a) The probability function of a discrete random variable $X$ is given by;

$$P(X=x)=\begin{cases} kx & ; \quad x=1,2,3,4,5 \\ 0 & ; \quad others \end{cases}.$$

(i) Determine the value of $k$.

(2 marks)

(ii) Find $P(1<X\leq 4)$.

(2 marks)

(iii) Calculate $E(X)$.

(3 marks)

(iv) Compute $Var(X)$.

(3 marks)

(b) The continuous random variable $X$ has a probability density function given by;

$$f(x)=\begin{cases} kx(1-x)^2 & ; \quad 0\leq x\leq 3 \\ 0 & ; \quad otherwise \end{cases}.$$

(i) Show that $k=\dfrac{4}{27}$.

(4 marks)

(ii) Determine $P(1\leq X\leq 3)$.

(3 marks)

(iii) Find $E(X)$.

(3 marks)

**Q5** (a) Suppose you are conducting an experiment in which you flip a coin. The coin has a probability, $p$ of landing on heads, where $p=0.4$. You flip the coin 15 times.

(i) Define the random variable in this context.

(1 mark)

(ii) Calculate the probability of getting exactly 8 heads in 15 flips.

(3 marks)

(iii) Determine the mean and standard deviation of the number of heads.

(4 marks)

(b) The average number of licenced drivers involved in at least one car accident in any given year is 6. Assume that the number of licenced drivers involved in at least one car accident follows a Poisson distribution. Find the probability that;

(i) only three will be involved in at least one accident in any given year. (2 marks)

(ii) at most two will be involved in two accidents in any given year. (4 marks)

(c) Suppose that women aged 18 – 24 years old, systolic blood pressure (in mm Hg) are normally distributed with mean of 114.8 and standard deviation of 13.1. If a woman between ages of 18 – 24 is randomly selected, find the probability that her systolic blood pressure is;

(i) above 120 mm Hg. (3 marks)

(ii) at most 130 mm Hg. (3 marks)

**- END OF QUESTIONS -**

## FINAL EXAMINATION

SEMESTER / SESSION : SEM I 2023/2024 PROGRAMME CODE : DAK
COURSE NAME : TECHNICAL MATHEMATICS III COURSE CODE : DAS21002

### Table 1: Vector

| | |
|---|---|
| $\lvert\mathbf{u}\rvert = \sqrt{a^2+b^2+c^2}$ | $\hat{\mathbf{u}} = \dfrac{\mathbf{u}}{\lvert\mathbf{u}\rvert}$ |
| $\mathbf{u}\bullet\mathbf{v} = u_1v_1+u_2v_2+u_3v_3$ | $\mathbf{u}\bullet\mathbf{v} = \lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert\cos\theta$ |
| $\theta = \cos^{-1}\left(\dfrac{\mathbf{u}\bullet\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}\right)$ | $A = \dfrac{1}{2}\lvert\mathbf{u}\times\mathbf{v}\rvert$ |
| $\mathbf{u}\times\mathbf{v} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\end{vmatrix}$ | $x = x_0 + a_1t$<br>$y = y_0 + a_2t$<br>$z = z_0 + a_3t$ |
| $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$ | |

### Table 2: Complex Number

| | |
|---|---|
| $z = a + bi$<br>$\bar{z} = a - bi$ | $z = r(\cos\theta + i\sin\theta)$ |
| $r = \sqrt{a^2+b^2}$ | $\theta = \tan^{-1}\left(\dfrac{b}{a}\right)$ |
| $z_1z_2 = r_1r_2[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)]$ | |
| $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)]$ | |
| $z = re^{i\theta}$ | $z^n = r^ne^{in\theta}$ |
| $z^{\frac{1}{n}} = r^{\frac{1}{n}}e^{\left(\frac{\theta+2k\pi}{n}\right)i}$ | $z^n = r^n[\cos n\theta + i\sin n\theta]$ |
| $z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\dfrac{\theta+2k\pi}{n} + i\sin\dfrac{\theta+2k\pi}{n}\right)$ ; $k = 0,1,2,\ldots,n$ | |

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### Table 3: Probability

| | |
|---|---|
| $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ | $P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$ |

### Table 4: Descriptive Statistics

| | |
|---|---|
| $\mu = \dfrac{\sum_{i=1}^{n} x_i}{N}$ | $\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum f}$ |
| $\sigma^2 = \dfrac{\sum_{i=1}^{n} (x_i - \mu)^2}{N}$ | $s^2 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$ |
| $s^2 = \dfrac{1}{\Sigma f - 1}\left[\sum_{i=1}^{n} f_i x_i^{\,2} - \dfrac{(\Sigma f_i x_i)^2}{\Sigma f}\right]$ | |
| $M = L_m + C \times \left(\dfrac{\frac{n}{2} - F_{m-1}}{f_m}\right)$ | $M_0 = L + C \times \left(\dfrac{d_b}{d_b + d_a}\right)$ |

### Table 5: Probability Distribution

| |
|---|
| Binomial $X \sim B(n,p) = \binom{n}{r} p^r (1-p)^{n-r}$ for $n = 0,1,\ldots,n$ |
| Poisson $X \sim P_0(\mu) = \dfrac{e^{-\mu}\mu^r}{r!}$ for $\mu = 0,1,2 \ldots$ |
| Normal $X \sim N(\mu, \sigma^2)$ , $f(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\left[\frac{(x-\mu)^2}{2\sigma^2}\right]}$ |
| Standard Normal $Z \sim N(0,1)$, $f(z) = \dfrac{1}{\sqrt{2\pi}} e^{-\left[\frac{z^2}{2}\right]}$, $z = \dfrac{x-\mu}{\sigma}$ |

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**Table 6: Random Variables**

| | |
|---|---|
| $\sum_{i=-\infty}^{\infty} p(x_i) = 1$ | $\int_{-\infty}^{\infty} f(x)\ dx = 1$ |
| $E(X) = \sum_{\forall x} xp(x)$ | $E(X) = \int_{-\infty}^{\infty} xp(x)\ dx$ |
| $Var(X) = E(X^2) - [E(X)]^2$ | |