CONFIDENTIAL

Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2023/2024

COURSE NAME	:	ALGEBRA
COURSE CODE	:	DAE13003 / DAM13003 / DAS10103
PROGRAMME CODE	:	DAE / DAM / DAU
EXAMINATION DATE	:	JANUARY / FEBRUARY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 Let $z = x + iy$ be a nonzero complex number.

(a) Show that $\dfrac{\overline{z}}{|z|^2} = \dfrac{1}{z}$.

(6 marks)

(b) Given that $z\overline{z} + 2iz = 3 + 4i$. Find the value of x and y.

(7 marks)

(c) Hence, apply De Moivre's Theorem to find all the cube roots of z.

(7 marks)

Q2 (a) Given $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$. Show that:

$$|\mathbf{u} \times \mathbf{v}|^2 = |\mathbf{u}|^2 |\mathbf{v}|^2 - (\mathbf{u} \bullet \mathbf{v})^2.$$

(7 marks)

(b) Given three points $A = (1, 2, 4)$, $B = (0, -1, -3)$ and $C = (3, 2, -1)$. Calculate the area of a triangle ABC.

(7 marks)

(c) Find the parametric and symmetric equations of the straight line that passes through the points $P = (-2, -4, 4)$ and $Q = (-4, 2, -3)$.

(6 marks)

Q3 (a) By using Binomial theorem, expand $\dfrac{1}{\left(1 - \dfrac{x}{2}\right)^4}$ in ascending power of x up to the term x^3.

(5 marks)

(b) Given that $\sin A = \dfrac{3}{5}$ where $90° \le A \le 180°$ and $\cot B = \dfrac{5}{12}$ where $180° \le B \le 270°$. Find:

(i) $\tan A + 3\sin B$.

(4 marks)

(ii) $\sec B - \cos A$.

(2 marks)

(c) Show that $\cos\theta(\tan\theta + \cot\theta) = \csc\theta$.

(4 marks)

(d) Solve $\cos x - \sin x \tan x + 1 = 0$ for $0° \leq x \leq 360°$.

(5 marks)

Q4 (a) Divide $-3x^3 + 4x^2 - x + 5$ by $x + 1$ using long division method.

(5 marks)

(b) Solve the following logarithmic equation:

$$\log_8(17x^2 - 8x - 40) = \frac{1}{3} + 2\log_8(3x - 2).$$

(5 marks)

(c) Express the following as a sum of partial fractions:

$$\frac{5x^2 + 4x - 7}{(2x^2 + x)(x - 4)}.$$

(5 marks)

(d) Find the root of $2x - 5\cos x = 0$, in $[0.5, 1.5]$ by using Secant method. Iterate until $|f(x_i)| < \varepsilon = 0.005$. Set your calculator in radian mode and show all your calculations in three decimal places.

(5 marks)

Q5 (a) Given that $A = \begin{bmatrix} -3 & 1 & 2 \\ -1 & 3 & 4 \\ 2 & 1 & -1 \end{bmatrix}$. Find:

(i) Determinant, $|A|$.

(3 marks)

(ii) The inverse matrix of A by using formula, $A^{-1} = \frac{1}{|A|} Adj(A)$.

(7 marks)

(b) By using the Gauss – Jordan elimination method, solve the following system of linear equations.

$$x + 2y - 5z = 9$$
$$-2x + y - 2z = -11.$$
$$3y - z = -4$$

Do the following operations in order:

$$R_2 + 2R_1,\ \frac{1}{5}R_2,\ R_1 - 2R_2,\ R_3 - 3R_2,\ \frac{5}{31}R_3,\ R_1 + \frac{1}{5}R_3,\ R_2 + \frac{12}{5}R_3.$$

(10 marks)

- END OF QUESTIONS -

LIST OF FORMULA

Polynomials

$$\log_a x = \frac{\log_a x}{\log_a b} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c,$$

$$x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

Partial Fraction

$\frac{P(x)}{(x+b)(x+c)} = \frac{A}{x+b} + \frac{B}{x+c}$
$\frac{P(x)}{x(x+b)(x+c)} = \frac{A}{x} + \frac{B}{x+b} + \frac{C}{x+c}$
$\frac{P(x)}{(x+b)^2} = \frac{A}{x+b} + \frac{B}{(x+b)^2}$
$\frac{P(x)}{(x+b)(x^2+c)} = \frac{A}{(x+b)} + \frac{Bx+C}{(x^2+c)}$

Sequence and Series

$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \frac{n(n-1)(n-2)(n-3)}{4!}b^4 + \cdots$$

Trigonometry

$\sin^2 x + \cos^2 x = 1$, $\tan^2 x + 1 = \sec^2 x$, $1 + \cot^2 x = \csc^2 x$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$, $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$, $\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

Vector

$\lvert\mathbf{u}\rvert = \sqrt{a^2+b^2+c^2}$	$\hat{\mathbf{u}} = \dfrac{\mathbf{u}}{\lvert\mathbf{u}\rvert}$
$\mathbf{u}\bullet\mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$	$\mathbf{u}\bullet\mathbf{v} = \lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert\cos\theta$
$\theta = \cos^{-1}\left(\dfrac{\mathbf{u}\bullet\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}\right)$	$A = \dfrac{1}{2}\lvert\mathbf{u}\times\mathbf{v}\rvert$
$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$	$\dfrac{x-x_0}{a_1} = \dfrac{y-y_0}{a_2} = \dfrac{z-z_0}{a_3}$
$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$	

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Complex Number

$z = a + bi$ $\bar{z} = a - bi$	$z = r(\cos\theta + i\sin\theta)$
$\lvert z \rvert = r = \sqrt{a^2 + b^2}$	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$
$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$	
$z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{\left(\frac{\theta + 2k\pi}{n}\right)i}$	
$z^n = r^n\left[\cos n\theta + i\sin n\theta\right]$	
$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{\theta + 2k\pi}{2} + i\sin\frac{\theta + 2k\pi}{2}\right)$	

Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$